Less competition, more meritocracy?*

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Abstract

Uncompetitive contests for grades, promotions, retention, and job assignments, which feature lax standards and limited candidate pools, are often criticized for being unmeritocratic. We show that, when contestants are strategic, lax standards and exclusivity can make selection more meritocratic. When many contestants compete for a few promotions, strategic contestants adopt high-risk strategies. Risk-taking reduces the correlation between performance and ability. Through reducing the effects of risk-taking, “Peter-Principle” promotion policies, which entail promoting some contestants that are unlikely to be worthy, can increase the overall correlation between selection and ability, and thus further the goal of meritocratic selection.

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1 Introduction

Competitions to identify and select “the best and the brightest”—e.g., educational tests, worker performance evaluations, league-table rankings of mutual funds, are a pervasive feature of modern life. Given the growing labor-income share of “working rich” (Piketty, 2005), an elite selected through meritocratic selection contests, and their dominant position in multinational corporations and global institutions (Brezis and Temin, 2008), such selection contests clearly have profound social and economic effects.

The design of selection contests is frequently shaped by the perspective that competition and high standards are fundamental features of meritocratic selection or even its defining characteristic (cf. Frost, 2017). In fact, “meritocratic society” is sometimes even treated as a synonym for “competitive society” (cf. Ekins, 2014). This paper, however, shows that, when contestants are strategic, meritocratic selection can often be furthered by anti-competitive policies such as low selection bars and restricted candidate fields.

These implications are developed in a parsimonious model of contest design. In the model, n contestants compete for selection. The number of contestants selected is determined by the contest’s selection quota, m. Contestants prefer selection to deselection.

Each contestant is endowed with ability: strong contestants have high ability and weak contestants have low ability. Ability is private information. Ability represents the characteristics of the contestant valued by the contest designer. Selection is based on performance in a contest (e.g., portfolio returns, examination scores). The expected performance of a contestant, which we term the contestant’s performance capacity, is fixed by the contestant’s ability. However, contestants can take risks, i.e., submit performance that is a mean-preserving spread of capacity.¹ Contestants are selected based on the ranking of their performance relative to rival contestants.

The design of the contest, i.e., the number of contestants, n, and the number of quota places, m, is entrusted to a contest designer. In contrast to the existing literature on the effects of competitiveness on effort, our focus is on the effect of competitiveness on selection efficiency.²

¹Risk-taking can take many forms depending on the nature of the competition: in mutual-fund manager competitions, increasing the unsystematic risk exposure (Chevalier and Ellison, 1997; Khorana, 2001; Kaniel and Parham, 2017), in weight-lifting competitions, attempting a very heavy lift (Genakos and Paglierio, 2012), in sales contests, pursuing new customers rather than cultivating current customers.

Risk-taking strategies have been theoretically modeled as a mean-preserving spread of a fixed performance level in various contexts, such as fund manager competitions (Bell and Cover, 1980), sales contests (Gaba and Kalra, 1999), and R&D races (Dasgupta and Stiglitz, 1980; Klette and de Meza, 1986). In Online Appendix D, we show that the qualitative nature of our analysis extends to the cases where the mean constraint is not imposed directly on performance but on any increasing function of performance, which allows for mean performance to be decreasing in risk-taking.

²See Moldovanu and Sela (2001), Fang et al. (2019), and Olszewski and Siegel (2019) for the analyses of the effects of competitiveness on effort. Extending our framework to incorporate effort would not qualitatively affect our basic results. See Section 6.2. However, incorporating effort into the contest-design objective function would obscure one of the most novel conclusions of our analysis: uncompetitive contest designs can be motivated simply by the objective of furthering meritocracy.
In the baseline model, the designer’s objective function is symmetric, i.e., the gain from selecting a strong contestant equals the loss from selecting a weak contestant. The problem the designer faces is that ability is private information and thus is not directly observable. Contestant ability must be inferred from contestant performance. Contestant performance is affected by contestant risk-taking.

We first consider situations in which the number of contestants is fixed but the designer can vary the selection quota, e.g., a promotion contest within a firm in which the firm can choose the promotion rate. Our analysis shows that, when the expected quality of the contestant pool is not too low, “Peter-Principle” selection—choosing a large selection quota under which some selected candidates are likely to be weak—can be optimal. Although selecting these unworthy candidates per se reduces meritocratic designer welfare, a lax selection standard, by increasing the probability that weak contestants will be selected, reduces weak contestants’ incentives to adopt high-risk strategies to challenge strong contestants, thereby increasing the correlation between performance and ability. When the expected quality of the contestant pool is not too low, the latter effect can be sufficiently large such that a lax selection standard maximizes designer welfare.

This result contrasts sharply with results in other papers that verify the optimality of “over-promotion” polices (e.g., Prendergast, 1992; Fairburn and Malcomson, 2001; Gürtler and Kräkel, 2010). These papers consider situations in which employers are willing to make hiring less meritocratic, through over-promotion, in order to increase the incentive efficiency of compensation. We examine whether over-promotion can be motivated simply by the objective of making hiring more meritocratic.

When the expected quality of the contestant pool is sufficiently low, lax selection standards are likely to result in too many weak candidates selected and are thus not optimal. However, increasing selectivity by shrinking the quota increases risk-taking, which attenuates the correlation between performance and ability. Attenuation, given the expected low quality of the contestant pool, results in the expected quality of the best performing contestant being insufficient to justify selection. In this case, designer welfare is maximized by setting a zero quota. Consequently, risk-taking renders the contest mechanism an ineffective means of identifying able candidates out of a weak candidate pool. This ineffectiveness can be costly given many relative advantages of contests over other selection mechanisms.3

Next we consider the case where the selection quota is fixed but the designer can vary the number of contestants, e.g., a competition for a firm’s CEO position. Firms have only one CEO, so the selection quota is fixed. However, firms can vary the number of contestants either

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3A number of researchers have argued that contest selection is more advantageous than selection based on absolute performance when performance is hard to verify (Che and Gale, 2003) or affected by common time-varying shocks (Lazear and Rosen, 1981; Knoeber and Thurman, 1994), as well as when relative performance is easier to measure (Lazear and Rosen, 1981), when performance evaluation is delegated to lenient reviewers (Letina et al., 2019), and when firms have a strong preference for offering a fixed amount of total compensation to employees (Gürtler and Kräkel, 2010).
by excluding external candidates, through an “in-house” competition, or including external candidates, through an “open competition.” We show that, once the number of contestants increases beyond a threshold level, adding even more contestants does not increase the expected quality of selected contestants, and thus does not increase meritocratic welfare.

In contrast, in contest models that do not feature strategic risk-taking, increasing the number of contestants generally increases expected candidate quality. For example, Ryvkin (2010, Corollary 3.1) shows that, in selection contests where contestant performance equals ability plus an exogenous i.i.d. noise, expected winner ability is always increasing in contest size.\(^4\) In all-pay contests, where contestant performance equals contestant effort and effort costs depend on contestant ability, a contestant’s performance rank equals her ability rank if contestants are ex-ante homogeneous with ability being private information (Moldovanu and Sela, 2001), and approximates her ability rank if contest size is large (Olszewski and Siegel, 2016, 2019). Thus, in these contests, adding contestants also increases expected winner ability.\(^5\)

After developing the baseline results, we extend our baseline analysis in order to extend its scope of application. Our first extension considers asymmetric designer objective functions, functions that specify a gain from selecting strong contestants that need not equal the loss from selecting weak contestants. We show that the extended model’s predictions are qualitatively identical to the baseline model’s. We then show that this extension permits us to incorporate retention contests into our analysis. In a retention contest, the firm sets a quota for the number of workers retained. Workers who fill the quota are retained and the remainder are dismissed and replaced with new hires. We show that, in retention contests designed to optimize workforce quality, because of strategic risk-taking, when the number of workers is large and the expected quality of the incumbent worker pool is low, the fraction of workers dismissed can be far lower than the fraction of workers expected to have low ability.

Next, we extend the analysis to endogenous contestant capacity. We assume that both high- and low-ability contestants must invest effort to produce performance capacity (e.g., students studying in order to increase their capacity to perform in an examination). Total and marginal costs of capacity acquisition are lower for high-ability contestants. Contestants choose performance distributions that are mean-preserving spreads of their endogenously acquired capacity. We show that our quota-inflation and contest-size results are robust to endogenous capacity and that these results extend to situations in which the contest designer uses contests both for selection and for the provision of effort incentives.

Finally, we show that our contest-design analysis extends to situations in which contestant performance naturally features an upper bound, such as full marks in student examinations. The analysis from this extension implies that reducing the competitiveness of elite-university admissions by randomly allocating \(m\) admission places over the \(m' > m\) best performing ap-

\(^4\)While Ryvkin (2010) restricts the selection quota to one, his result can be easily extended to the case with an arbitrary fixed quota.

\(^5\)However, the objective in these papers is not to increase expected winner ability. Thus, the increase in winner quality induced by larger contest size need not imply the optimality of increasing contest size in these papers.
applicants, a mechanism proposed by Schwartz (2007), need not reduce the quality of admitted applicants. Schwartz (2007) argues that reducing competition can reduce the psychological costs imposed on students by highly selective university admissions. Our analysis suggests that society can obtain this benefit without sacrificing meritocracy.

In summary, our analysis predicts that, because of strategic risk-taking by contestants, meritocratic contest designers will frequently have no incentive to run inclusive contests and, when choosing promotion and reward schemes, will never choose schemes that are expected to reject worthy candidates but frequently adopt schemes that are expected to select unworthy candidates. Thus, when contestants are strategic, meritocracy is consistent with exclusive contests with lax selection standards.

Many observed contests seem to have these characteristics. Social promotions and lax grading by schools can be interpreted as awards to sub-marginal performance ranks. In our analysis, such policies can make selection more meritocratic. For the same reasons, motivational promotions by firms can be rationalized even when such “motivational promotions” have no motivational effect. In retention contests, where not being dismissed is the contest reward, our results predict that dismissal rates will be lower under strategic risk-taking than when the dismissal rate is fixed purely on the basis of the distribution of contestant ability. This conclusion appears to be consistent with empirical studies of dismissals in mutual funds.

Related literature

This paper studies the effect of risk on selection. Its principal departure from the extant literature is that it models “endogenous” risk, risk generated by contestant strategies rather than risk generated by an exogenous noise term that mediates the relationship between contestant ability and contestant performance. Endogenous risk-taking is modeled in a “fair-gambles framework,” in which contestants choose performance distributions that are mean-preserving spreads of their endowed capacity. The fair-gambles framework has been adopted in many studies of contests, including political campaigns (Myerson, 1993; Lizzeri, 1999), status contests (Robson, 1992; Becker et al., 2005; Ray and Robson, 2012), and fund manager competitions (Bell and Cover, 1980; Seel and Strack, 2013; Fang and Noe, 2016; Strack, 2016). Strategic risk-taking in competitive environments has also been documented empirically in a number of contexts (Chevalier and Ellison, 1997; Khorana, 2001; Bothner et al., 2007; Beaudoin and Swartz, 2010; Genakos and Pagliero, 2012) and in laboratory experiments utilizing professional subjects (Kirchler et al., 2018).

Lazear (2004) studies promotion in a model where risk is generated by exogenous noise affecting the performance/ability relation. His analysis, like ours, identifies a Peter-Principle

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6 For a discussion of social promotion in schools, see Jimerson et al. (2006). For a discussion of motivational promotion in the workplace, see Deeprose (2006).

7 Khorana (1996, Table 4) finds that only 14% of mutual-fund managers in the lowest performance decile are replaced despite the fact that, as Khorana (2001) documents, replacing low-performing managers improves mutual-fund returns.
effect: because a component of worker performance is produced by luck, i.e., a high realiza-
tion of a random noise term affecting contest performance, the expected future performance
of promoted workers is less than their performance in the promotion contest. However, the
implications of our analysis differ quite dramatically from Lazear (2004). In Lazear (2004), the
contest designer, realizing that part of contestant performance is the product of luck, adjusts
upward the performance cutoff required for promotion. Thus, in Lazear (2004), exogenous
risk leads to an increase in promotion standards relative to the no-noise case. In our analysis,
endogenous risk leads the designer to lower standards by expanding the selection quota relative
to the quota which would have been selected in the absence of risk.

Like Lazear (2004), Ryvkin and Ortmann (2008) and Ryvkin (2010) also consider the effect
of exogenous risk on selection. These papers fix the selection quota at one, and thus, unlike our
paper and Lazear (2004), they do not address the effect of risk on the selection quota. However,
they do consider the effect of expanding the contestant pool. A common conclusion they reach
is that expanding the contestant pool always strictly increases the expected ability of the winner.
In contrast, we show that, when contestants are strategic risk-takers, pool expansion beyond a
threshold number of contestants never strictly increases the expected ability of the winner(s).

Like Ryvkin and Ortmann (2008) and Ryvkin (2010), Hvid and Kristiansen (2003) also
fixes the selection quota at one. However, Hvid and Kristiansen (2003) does take into account
strategic risk-taking. Hvid and Kristiansen find an example in which the expected ability of the
winner is not uniformly increasing in the size of the contestant pool. This example is consistent
with the idea that contestant risk-taking can nullify the gains from quota expansion.

However, in Hvid and Kristiansen (2003), because contestants can only randomize be-
tween a safe and an exogenously specified risk-taking strategy, once all contestants switch to
the risk-taking strategy, it is not possible to accommodate further pool expansion by further
increasing risk. Thus, in Hvid and Kristiansen (2003), when pool size reaches the threshold
at which all contestants play the risk-taking strategy, the effect of further pool expansion on
winner quality is the same as the effect of pool expansion in exogenous-noise settings—i.e., it
always increases the expected winner quality. As our paper shows, this result no longer holds
when risk-taking strategies are not exogenously specified.

More generally, our paper is related to research showing that meritocracy can be furthered
by seemingly unmeritocratic policies. Meyer (1991) and Kawamura and Moreno de Barreda
(2014) find that biasing the contest selection mechanism toward certain contestants can increase
selection efficiency.\footnote{In a paper titled “The Limits of Meritocracy,” Morgan et al. (2018) model contests with exogenous noise; we model contests without exogenous noise; they define “meritocratic” to mean low contest noise; the designer’s objective is effort maximization not meritocratic selection. The questions addressed in Morgan et al. (2018) are interesting and important. However, the only commonalities between Morgan et al. (2018) and our paper are that both papers model contests and both use the term “meritocracy.”}
2 An example

In this section, we provide an example that aims to capture the fundamental intuition underlying our analysis. The example is developed heuristically, and precise statements and discussion of the model’s assumptions are deferred to the following sections. The example we consider is the problem faced by an administrator who has been tasked with setting the grade distribution for an examination. The exam will be taken by $n \geq 2$ students. There are two types of students: more able strong students, $S$, and less able weak students, $W$. A student’s type is the student’s private information and is independent of the types of the other students. Each student is strong, $S$, with probability $\theta$, and weak, $W$, with probability $1 - \theta$. Thus, although students know the number of students taking the exam, and know their own type, they do not know how many other exam-takers are strong. The administrator also does not know the number of strong students taking the exam. The distribution of student quality and the number of students taking the exam are common knowledge.

Expected exam performance is fixed by a student’s type. Strong students receive a mark of $\mu_S$ in expectation while weak students receive a mark of $\mu_W$ in expectation, $0 < \mu_W < \mu_S \leq \bar{x}$, where $\bar{x}$ represents full marks on the examination. Although a student’s expected mark is fixed by the student’s type, every student stands a chance of receiving a high or even full marks by taking risks. In practice, students can choose the intensity with which they revise potential exam questions. Thus, one high-risk strategy for a student is concentrating her limited capacity on studying (or memorizing) answers to a subset of potential questions. Using this strategy, a student will receive high marks if the exam questions happen to be drawn from this subset. Of course, if the student picks the wrong subset, her marks will be quite low.

We assume that a student can choose any mark distribution over $[0, \bar{x}]$ subject to the constraint that her expected mark equals $\mu_S$, if she is strong, and equals $\mu_W$, if she is weak. For example, if $\bar{x} = 100$ represents full marks, and a student’s expected mark equals 50, then a uniform mark distribution on $[0, 100]$ is feasible to this student, as is the mark distribution that produces full marks with probability $1/2$ and zero marks with probability $1/2$. We impose the zero and $\bar{x}$ bounds on performance because the context of this example is an examination. Although our analysis accommodates such bounds, they are not required to obtain our results, provided that a lower bound exists.\footnote{See Section 3 and Section 6.3 for further discussion.}

Based on exam performance, students are awarded an $A$ or a $B$. Each student prefers receiving an $A$ to receiving a $B$. The exam is “graded on a curve” which is set before the exam is taken and is known by the students. The curve specifies the proportions of $A$ and $B$ grades, or equivalently, given that the number of students is fixed at $n$, the curve specifies a quota, $m$, for the number of $A$ grades awarded. Grade awards are based on relative performance: if the curve is set at $m$, the $m$ students with the highest marks receive $A$ grades and the rest receive $B$ grades. Ties are broken by fair randomization.
The administrator is tasked with setting the grading curve, $m$. The administrator’s only objective is meritocratic grade assignment, assigning $A$ grades to strong students and $B$ grades to weak students. Because giving a weak student a $B$ grade is equivalent to not giving a weak student an $A$ grade, this objective is equivalent to assigning $A$ grades to strong students and not assigning $A$ grades to weak students. We employ a simple administrator welfare function consistent with this objective: the administrator sets the curve to maximize his welfare, $u$, where

$$u = \mathbb{E}[^\#\text{Strong students receiving an } A - ^\#\text{Weak students receiving an } A].$$

In this example, we assume that the parameters take the values $n = 20$, $\theta = 1/2$, $\mu_S = \bar{x} = 100$, and $\mu_W = 50$. Under these assumptions, there are 20 competing students, each of whom is equally likely to be strong or weak. Because, in this example, $\mu_S = \bar{x}$, a strong student always receives full marks, 100. A weak student receives only half marks, 50, in expectation (but can possibly receive full marks by taking risks).

**Non-strategic students** If weak students do not take risks and, hence, always receive half marks, strong students, who always receive full marks, will always outperform weak students and, thus, will always have priority over weak students, i.e., all strong students will receive an $A$ before any weak student receives an $A$. When the administrator sets the curve, $m$, the administrator does not know the number of strong students taking the exam but knows that each of the $n = 20$ students is strong with probability $\theta = 1/2$. Thus, the number of strong students is Binomially distributed with parameters $n = 20$ and $\theta = 1/2$.

Given the administrator’s objective, it is optimal to set the number of $A$ grades such that the lowest-ranked student receiving an $A$ is more likely to be strong than weak and the highest-ranked student receiving a $B$ is more likely to be weak than strong. Thus, it is optimal for the administrator to set the curve, $m$, such that the $m$-th highest-ability student has a probability of being strong no less than $1/2$ and the $(m+1)$-th highest-ability student has a probability of being strong no greater than $1/2$, i.e., it is optimal to set $m$ equal to the median of the Binomial$(20, 1/2)$ distribution. Because the median of the Binomial$(20, 1/2)$ distribution equals its mean, $20 \times (1/2) = 10$, if weak students do not take risks, it is optimal for the administrator to set the curve at $m = 10$, awarding an $A$ to top 10 exam performers and a $B$ to the rest of the exam-takers.

**Strategic students** What happens if students are strategic? We base our analysis on symmetric equilibria, in which students of the same type play the same strategy. We refer to equilibria in which weak students concede to (i.e., have zero probability of besting) strong students as *concession equilibria* and equilibria in which weak students challenge (i.e., have a positive probability of besting) strong students as *challenge equilibria*. Clearly, any symmetric equilibrium must be either a concession equilibrium or a challenge equilibrium. In this example, strong students’ performance is fixed at full marks. Thus, in the example, concession equilibria refer to equilibria in which weak students place no point mass on full marks while challenge equilibria refer to equilibria in which weak students place point mass on full marks.
Because the expected number of students receiving an A must equal $m$, in any symmetric equilibrium, the probability of receiving an A for a weak (strong) student, $p_W$ ($p_S$), must satisfy

$$m = n (\theta p_S + (1 - \theta) p_W).$$

(2)

Let $\tilde{S}_n$ be the number of strong students out of the $n$ students. Note that the number of strong students receiving an A cannot exceed the number of strong students, $\tilde{S}_n$, or the number of A grades, $m$, i.e.,

$$\#\text{Strong students receiving an A} \leq \min[\tilde{S}_n, m].$$

(3)

Taking expectations on both sides of (3) while noting that the expected number of strong students receiving an A equals $n \theta p_S$ implies that

$$n \theta p_S \leq \mathbb{E}\left[\min[\tilde{S}_n, m]\right].$$

(4)

Equations (2) and (4) imply that

$$m \leq \mathbb{E}\left[\min[\tilde{S}_n, m]\right] + n (1 - \theta) p_W.$$

(5)

Equation (5) yields the following lower bound, $p_C^W$, on a weak student’s probability of receiving an A in any symmetric equilibrium:

$$p_W \geq p_C^W := \frac{m - \mathbb{E}\left[\min[\tilde{S}_n, m]\right]}{n (1 - \theta)} = \frac{\mathbb{E}\left[m - \min[\tilde{S}_n, m]\right]}{n (1 - \theta)} = \frac{\mathbb{E}\left[\max[m - \tilde{S}_n, 0]\right]}{n (1 - \theta)} = \frac{1}{n} \sum_{s=0}^{m-1} \left(\begin{array}{c} n \\ s \end{array}\right) \theta^s (1 - \theta)^{n-s-1} (m - s),$$

(6)

where the last equality follows from the fact that $\tilde{S}_n \sim \text{Binomial}(n, \theta)$. Because equation (4) holds with equality if and only if weak students concede to strong students, the lower bound on $p_W$ given by $p_C^W$ in equation (6) is attained in and only in concession equilibria. Thus,

$$p_W = p_C^W.$$

(7)

Note that, in any symmetric equilibrium, a weak student’s probability of receiving an A by playing her equilibrium strategy must be no less than her probability of receiving an A by playing a risk-taking strategy that gives her a strong type’s mark distribution with probability $\mu_W/\mu_S$ and zero marks with the complementary probability. This risk-taking strategy produces an expected mark equal to $\mu_W$ and is thus feasible to a weak student. This strategy also gives the weak student a strong type’s probability of receiving an A, $p_S$, with probability $\mu_W/\mu_S$. Thus, this strategy gives the weak student a probability of receiving an A equaling at least $(\mu_W/\mu_S) p_S$, which must be no greater than the weak student’s probability of receiving an A by playing her equilibrium strategy, $p_W$, i.e.,

$$p_W \geq \frac{\mu_W}{\mu_S} p_S.$$

(8)

Equation (8) implies that $p_S \leq (\mu_S/\mu_W) p_W$. This fact, combined with equation (2), implies that

$$m \leq n \left( \theta \frac{\mu_S}{\mu_W} p_W + (1 - \theta) p_W \right) = p_W \frac{n}{\mu_W} (\theta \mu_S + (1 - \theta) \mu_W).$$

(9)
Equation (9) implies that a weak student’s probability of receiving an A has the following lower bound, \( p^G_o \), in any symmetric equilibrium:

\[
p_W \geq p^G_o := \frac{m}{n} \left( \frac{\mu_w}{\theta \mu_S + (1 - \theta) \mu_W} \right).
\]  

(10)

Equations (7) and (10) imply that

concession equilibria exist only if \( p^C_o \geq p^G_o \).

(11)

As we will verify in later sections, the necessary condition given by equation (11) is also sufficient for the existence of concession equilibria and, if a concession equilibrium does not exist, a challenge equilibrium exists.\(^\text{10}\) Thus, when \( p^C_o < p^G_o \), only challenge equilibria exist.

A simple computation, using equations (6) and (10) and the example’s parameters, shows that \( p^C_o < p^G_o \) is equivalent to \( m \) being an integer satisfying \( 1 \leq m \leq 14 \). Because, when \( 1 \leq m \leq 14 \), only challenge equilibria exist, if the grading curve specifies a quota for A grades, \( m \), less than 15, weak students will challenge strong students by playing a high-risk strategy that produces some chance of attaining full marks.

Now consider the implications of student strategic behavior for the administrator setting the curve. Formalizing the definition of administrator welfare function given by equation (1), shows that the administrator’s welfare is given by

\[
n \theta p_S - n (1 - \theta) p_W.
\]

Equation (2) implies that

\[
n \theta p_S = m - n (1 - \theta) p_W.
\]  

(12)

Consequently, the administrator’s welfare function can be expressed as follows:

\[
u(m) = m - 2 n (1 - \theta) p_W.
\]  

(13)

Equations (10) and (13) imply that

\[
u(m) \leq m \left( \frac{\theta \mu_S - (1 - \theta) \mu_W}{\theta \mu_S + (1 - \theta) \mu_W} \right).
\]  

(14)

Substituting the example’s parameters into (14) reveals that, when \( 1 \leq m \leq 14 \), \( u(m) \leq 14/3 \).

If the administrator sets the curve at \( m = 15 \), the necessary and sufficient condition for the existence of concession equilibria, i.e., \( p^C_o \geq p^G_o \), is satisfied. Thus, when \( m = 15 \), \( p_W = p^C_o \). By equation (6) and the example’s parameters, when \( m = 15 \), \( p^C_o \approx 0.501 \). Hence, by equation (13), the example’s parameters, and the fact that \( p_W = p^C_o \) when \( m = 15 \), \( u(15) \approx 4.985 \). Consequently, \( u(15) > 14/3 \geq u(m) \) for \( 1 \leq m \leq 14 \). Therefore, the administrator strictly prefers setting the curve \( m = 15 \) to any curve \( m < 15 \). The example’s parameters, equations (6) and (13), and the fact that \( p_W = p^C_o \) when \( 16 \leq m < 20 \) reveal that \( u(15) > u(m) \) for \( 16 \leq m < \)

\(^\text{10}\)In Section 3, we will verify these conditions for our baseline model, where we do not impose an exogenous upper bound, \( \bar{x} \), on the support of any feasible performance distribution. In Section 6.3, we will show that imposing an upper bound, \( \bar{x} \), has no effect on any of these conditions or selection outcomes as long as \( \bar{x} \geq \mu_S \).
Thus, \( m = 15 \) is the optimal grading curve.\(^{12}\)

Recall that, when weak students do not act strategically, it is optimal for the administrator to set the curve at \( m = 10 \). Thus, our analysis reveals that the administrator responds to strategic risk-taking by “grade inflation.” Grade inflation comes at a cost. The median number of strong students is 10. Thus, when \( m = 15 \), the fact that weak students concede to strong students implies that the five students ranked between the 11th and the 15th highest (inclusive) are in fact more likely to be weak than to be strong and are thus not expected to merit an A grade.

However, this cost is dominated by the benefit of grade inflation: setting the soft \( m = 15 \) curve ensures that weak students have no incentive to take the high risks required to challenge strong students, and thereby better aligns exam marks with student ability. Equation (14), applied to the example’s parameters, shows that, when students act strategically, the administrator’s welfare under \( m = 10 \), the optimal quota when students are non-strategic, is bounded above by \( 10/3 \approx 3.333 \). As shown above, under \( m = 15 \), the optimal quota when students act strategically, the administrator’s welfare equals \( u(15) \approx 4.985 \). Thus, despite of the cost of selecting five students who are not expected to merit an A grade, grade inflation increases the administrator’s welfare at least by (roughly) 50%. As the example shows, our meritocratic administrator, whose only goal is awarding A grades to strong students and B grades to weak students, can benefit from choosing grading curves that are expected to award A grades to many students who do not merit an A grade. Reducing competition, by setting a soft curve, furthers meritocratic selection.\(^{13}\)

3 Risk-taking

3.1 Risk-taking selection contests

Consider a contest with \( n \geq 2 \) contestants; \( m \) of them will be selected to fill a place, and the remaining \( n - m \) contestants will be deselected and not receive a place, where \( 0 < m < n \). The number of places, \( m \), which we call the selection quota, and the number of contestants, \( n \), which we call contest size, are fixed before the contest and are common knowledge.

The contestants are of two possible types, \( t \): strong, S, and weak, W. Whether a contestant is strong or weak is determined by an independent draw from a Bernoulli distribution which assigns probability \( \theta \) to S, and probability \( 1 - \theta \) to W. A contestant’s type is the contestant’s private information.

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11 More specifically, the computation shows that \( u(16) \approx 3.997 \), \( u(17) \approx 3.000 \), \( u(18) \approx 2.000 \), and \( u(19) \approx 1.000 \).

12 It is obvious that \( u(0) = 0 \) and, given \( \theta = 1/2 \), \( u(20) = 0 \). Thus, the curve \( m = 15 \) is optimal even if the administrator has the option of giving the students the same grade regardless of exam performance.

13 This result sharply contrasts with Chan et al. (2007), which models grade inflation as an equilibrium outcome of a signaling model in which the sender is a university with private information about student quality, the signal is the grading curve, and the receivers are potential student employers. In Chan et al. (2007), the designer (the university administration) aims to maximize student compensation, not meritocratic grade assignment, and grade inflation makes selection less meritocratic.
Selection is based on performance in the contest. Each type-$t$ contestant can take risky activities in the contest that add noise to her otherwise fixed performance capacity $\mu_t > 0$, $t \in \{S, W\}$. Strong contestants have higher capacity to perform i.e., $\mu_S > \mu_W$. We assume that the performance of a type-$t$ contestant must satisfy the capacity constraint: the expected performance of each type-$t$ contestant equals her capacity, $\mu_t$, $t \in \{S, W\}$. The relative capacity of the contestants thus measures the extent to which contest performance varies with contestant type and thus the degree to which contest outcomes can reveal contestant types. The capacity constraint only restricts the first moment of the contestants’ performance distributions. In Online Appendix D, we show that our results extend to risk-taking contests with more general capacity constraints which restrict other moments, such as variance.

We assume that each contestant can choose any distribution of nonnegative performance subject to the capacity constraint. The assumption that the feasible range for the contestants’ performance distributions is $[0, \infty)$ is made largely to simplify the exposition of our results. In practice, the feasible performance range will be determined by the nature of the technology used to measure performance, e.g., in our examination example, it is not possible to earn less than 0 marks or more marks than full marks for the exam, so performance has exogenous upper and lower bounds. In a contest between mutual-fund managers, where performance is measured by returns, there is no exogenous upper bound on performance and the lower bound on performance is $-100\%$.

However, provided that the bounds on performance do not rule out risk-taking or make satisfaction of the capacity constraint impossible, these bounds have little qualitative effect on the analysis. A contest with a nonzero lower bound, $\underline{x}$, satisfying $-\infty < \underline{x} < \mu_W$ given capacities $\mu_t$, $t \in \{S, W\}$, is strategically equivalent to a contest with a zero lower bound given capacities $\mu_t - \underline{x}$. For this reason, without loss of generality, we normalize the lower bound to 0 in all of our analysis.

The upper bound on performance can affect the qualitative properties of equilibrium strategies. However, the focus of our analysis is on the effect of contest structure on selection, and, as we show in Section 6.3, having an exogenous upper bound, $\bar{x}$, on performance, $\bar{x} \geq \mu_S$, does not change contestants’ equilibrium selection probabilities.\(^{14}\)

Contestant performance distributions determine the outcome of the contest: each contestant’s realized performance is independently drawn from her performance distribution. The $m$ contestants with the highest realized performances are selected and the remaining contestants are deselected. Ties are broken by fair randomization. Contestants are expected utility maximizers who strictly prefer selection to deselection. Thus, given that risk-taking is costless, each contestant chooses a performance distribution to maximize her probability of winning a place given her rivals’ strategies and the contest’s parameters.

\(^{14}\)In Section 6.3, we also consider the effect of relaxing the capacity constraint to permit contestants to “burn” capacity when $\bar{x} < \mu_S$. 

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3.2 Equilibria

To determine the effect of contest design on meritocracy, we first need to characterize equilibrium contestant behavior. We focus on symmetric equilibria in which contestants of the same type all play the same strategy, i.e., each type-$t$ contestant chooses performance distribution $F_t$ with support $\text{Supp}_t$, $t \in \{S, W\}$.

A contestant’s probability-of-winning function maps the contestant’s realized performance, $x$, to her probability of being selected and is thus determined endogenously by her rivals’ strategies. Because each contestant faces the same distribution of rival types and strategies are symmetric, all of the contestants face the same probability-of-winning function, $P : \mathbb{R}_+ \to [0, 1]$. Let $\text{Supp} P$ be the support of $P$. Because $P$ increases and only increases at points in the support of at least one type’s performance distribution, we have

$$\text{Supp} P = \text{Supp}_W \cup \text{Supp}_S. \quad (15)$$

To facilitate our equilibrium derivation, we make the following claim:

**Claim 1.** In any symmetric equilibrium, the probability-of-winning function, $P$, is (a) continuous and (b) has an interval support, $[0, \hat{x}]$, where $\hat{x} < \infty$ is endogenous.

Because many authors have established properties analogous to Claim 1 in symmetric fair-gambles contests and symmetric all-pay auctions (Barut and Kovenock, 1998; Fang and Noe, 2016), our verification of Claim 1 is not very original. So, we defer this verification to Online Appendix A.\footnote{See the proof of Lemma 1 in Online Appendix A.} In contests where performance is exogenously bounded above by $\bar{x}$, where $\bar{x} \geq \mu_S$, e.g., in the case of a student examination we saw in Section 2, contestants may place point mass on the upper bound, $\bar{x}$, in a symmetric equilibrium. In these contests, the probability-of-winning function, $P$, may not be continuous at the upper bound. Nevertheless, as we show in Online Appendix A, excluding the upper bound, $P$ is continuous and has an interval support starting from 0.\footnote{See the proof of Proposition 3 in Online Appendix A.}

Equation (15) and Claim 1 immediately imply that

$$\text{Supp}_W \cup \text{Supp}_S = [0, \hat{x}]. \quad (16)$$

To initiate the equilibrium derivation based on Claim 1, a few definitions are required: for any two points $(x_1, p_1), (x_2, p_2)$ in $\mathbb{R}_+^2$, we define the interval between the points, $[(x_1, p_1), (x_2, p_2)]$ by

$$[(x_1, p_1), (x_2, p_2)] = \{\lambda (x_1, p_1) + (1 - \lambda) (x_2, p_2) : \lambda \in [0, 1]\}.$$

A gamble between performance levels $x'$ and $x''$ represents a performance distribution that randomizes between $x'$ and $x''$. A fair gamble between $x'$ and $x''$ for a contestant of type $t$ is a gamble between $x'$ and $x''$ with the property that the probability of choosing $x'$, $\pi$, satisfies $\pi x' + (1 - \pi) x'' = \mu_t$. Because fair gambles are feasible performance distributions, if performance
levels \( x' \) and \( x'' \) are in the support of the equilibrium performance distribution of type \( t \), then a type-\( t \) contestant’s payoff from a fair gamble between \( x' \) and \( x'' \) equals her equilibrium payoff.\(^{17}\) Next, note that, for performance levels \( x_1, x_2, \) and \( x_3 \) satisfying \( x_1 < \mu_t < x_2 \) and \( x_1 < \mu_t < x_3 \), if \( (P(x_3) - P(x_1))/(x_3 - x_1) < (P(x_2) - P(x_1))/(x_2 - x_1) \), the interval \([x_1, P(x_1)), (x_3, P(x_3))\] lies below the interval \([x_1, P(x_1)), (x_2, P(x_2))\]. Because \( \mu_t \in [x_1, x_2] \cap [x_1, x_3] \), this implies that a payoff to a type-\( t \) contestant from a fair gamble between \( x_1 \) and \( x_2 \) exceeds the payoff from a fair gamble between \( x_1 \) and \( x_3 \). This result is illustrated by Figure 1.

\[ \text{Figure 1: Fair gambles and best replies.} \text{ In the figure, for a contestant of type } t \in \{S,W\}, \text{ the payoff from a fair gamble between } x_1 \text{ and } x_3, \text{ given by the intersection of the dashed line and the interval } [(x_1, P(x_1)), (x_3, P(x_3))], \text{ yields a lower payoff than a fair gamble between } x_1 \text{ and } x_2, \text{ given by the intersection of the dashed line and the interval } [(x_1, P(x_1)), (x_2, P(x_2))]. \]

Because all fair gambles in the support of a type-\( t \) contestant’s performance distribution must produce the same payoff, \( x_3 \) cannot be in the support of \( t \)’s performance distribution if \( x_1 \) and \( x_2 \) are in its support. Similarly, if \( (P(x_3) - P(x_1))/(x_3 - x_1) > (P(x_2) - P(x_1))/(x_2 - x_1) \), \( x_2 \) cannot be in the support of \( t \)’s performance distribution if \( x_1 \) and \( x_3 \) are in the support. Thus, the slope of the line joining any two points \((x_1, P(x_1))\) and \((x_2, P(x_2))\) in the support of \( t \)’s performance distribution is constant, and hence all performance/probability-of-winning pairs \((x, P(x))\) such that \( x \in \text{Supp}_t \) are collinear.\(^{18}\)

Now consider the implications of collinearity for equilibrium behavior. First, note that the definition of the support of a distribution function, and Claim 1, imply that, \( \text{Supp}_W \cup \text{Supp}_S \) is a connected closed set. The connectedness of \( \text{Supp}_W \cup \text{Supp}_S \) and the fact that, by definition, both \( \text{Supp}_W \) and \( \text{Supp}_S \) are closed imply that the intersection, \( \text{Supp}_W \cap \text{Supp}_S \), is not empty.

If the intersection consists of a single point, say \( \bar{x} \), then, because \( \mu_S > \mu_W \), it must be the case that \( \text{Supp}_W \) and \( \text{Supp}_S \) are adjacent closed intervals meeting at \( \bar{x} \), with \( \text{Supp}_S \) lying above \( \text{Supp}_W \). In this case, the equilibrium is a concession equilibrium.\(^{19}\) The collinearity condition then implies that \( P \) is linear over \( \text{Supp}_W \) and over \( \text{Supp}_S \) with a possible kink at \( \bar{x} \). The slope of \( P \) over \( \text{Supp}_W \) cannot be less than the slope of \( P \) over \( \text{Supp}_S \). Were the slope to be strictly lower, \( P \) would be convex and nonlinear over its support \( \text{Supp}_W \cup \text{Supp}_S = [0, \bar{x}] \). In this case, by Jensen’s inequality, choosing a fair gamble between 0 and \( \bar{x} \) would be a strictly dominant

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\(^{17}\)The continuity of \( P \) implies that the set of optimal fair gambles is closed. Thus, the support of a contestant’s equilibrium performance distribution is contained in the set of optimal fair gambles for the contestant.

\(^{18}\)Collinearity for the single performance level where \( x = \mu_t \) follows from the continuity of \( P \).

\(^{19}\)The continuity of \( P \) implies that no contestant places any mass on \( \bar{x} \). Thus, a weak contestant cannot best strong contestants even if the weak contestant’s performance reaches the lower bound of \( \text{Supp}_S \), \( \bar{x} \).
strategy for each contestant and thus, no performance level between 0 and $\bar{x}$ could lie in the support of either type’s performance distribution, contradicting equation (16). Because, by Claim 1, $P$ is continuous, $P(0) = 0$. These observations imply that the probability-of-winning function in any concession equilibrium must satisfy the following conditions: $\text{Supp}_W = [0, \bar{x}]$, $\text{Supp}_S = [\bar{x}, \hat{x}]$, $P$ meets the origin and is weakly concave, strictly increasing over $[0, \bar{x}]$, linear over $[0, \bar{x}]$, and linear over $[\bar{x}, \hat{x}]$. Figure 2.A illustrates the satisfaction of these conditions.

If the intersection, $\text{Supp}_W \cap \text{Supp}_S$, contains more than one point, say at least the distinct points $x'$ and $x''$, then the probability that a weak contestant’s performance exceeds a strong contestant’s is positive. In this case, the equilibrium is a challenge equilibrium. The collinearity condition then implies that all pairs $(x, P(x))$ such that $x \in \text{Supp}_t$, $t \in \{S, W\}$, are collinear with the two points, $(x', P(x'))$ and $(x'', P(x''))$. Thus, all pairs $(x, P(x))$ such that $x \in \text{Supp}_W \cup \text{Supp}_S$ are collinear, which, by equations (15) and (16), implies that $P$ is linear over its support, $[0, \bar{x}]$. The continuity of $P$ given by Claim 1 implies that $P(0) = 0$. These observations imply that any challenge equilibrium must satisfy the following conditions: there exists $\hat{x}$ such that $\text{Supp}_W \cup \text{Supp}_S = [0, \hat{x}]$, $P$ is linear over $[0, \hat{x}]$ and meets the origin. Figure 2.B illustrates the satisfaction of these conditions.

![Figure 2: The possible forms of the probability-of-winning function, $P$, in a symmetric equilibrium.](image)

Using the properties of the probability-of-winning function, $P$, we first compute contestant payoffs in challenge equilibria. In challenge equilibria, $P$ is concave, which implies, by Jensen’s inequality, that choosing a deterministic performance level equal to the capacity level is a weakly optimal strategy for each type. Hence, we can evaluate each type’s probability of winning in challenge equilibria simply by evaluating $P$ at the type’s capacity. Thus, given that, in challenge equilibria, $P$ is linear over its support and meets the origin, it must be that, in challenge equilibria, the ratio between strong and weak types’ probabilities of winning, $p_s/p_w$, equals their capacity ratio, $\mu_s/\mu_w$. Thus, in challenge equilibria, equation (8) holds with equality. This fact, together with identity (2), implies that

$$p_w = p_o^G,$$

(17)

where $p_o^G$ is defined in equation (10).
Recall that, in Section 2, we derived two lower bounds, $p_C^o$ and $p_G^o$, on $p_W$ in any symmetric equilibrium, with $p_W = p_C^o$ in concession equilibria. While we assumed an exogenous upper bound, $\bar{x} = \mu_S$, on performance, the derivation does not depend on the upper bound, $\bar{x}$. Thus, equations (6), (7), (10), and (11) also hold absent the upper bound, $\bar{x}$. By equations (6), (7), and (17),

\[ p_W = \max\left[p_C^o, p_G^o\right]. \]

Challenge equilibria exist only if $p_G^o > p_C^o$. (18)

Because the necessary condition for the existence of concession equilibria, (11), and the necessary condition for the existence of challenge equilibria, (18), are complementary and because, as we show in Online Appendix B, a symmetric equilibrium always exists, these necessary conditions are also sufficient conditions. Thus, equations (2), (7), and (17) imply our first result. We relegate all of the formal proofs to the Online Appendices.

**Lemma 1.** Concession (challenge) equilibria exist if and only if $p_C^o \geq p_G^o \left(p_G^o > p_C^o\right)$, where $p_C^o$ and $p_G^o$ are given by equations (6) and (10) respectively. A weak contestant’s equilibrium probability of winning, $p_W$, is given by

\[ p_W = \max\left[p_C^o, p_G^o\right]. \]

A strong contestant’s equilibrium probability of winning, $p_S$, is determined by $p_W$ through equation (2).

Because Lemma 1 provides a characterization of equilibria that is sufficient for analyzing the effect of strategic risk-taking on meritocratic selection, we defer the construction of equilibrium performance distributions to Online Appendix B.\(^{20}\)

## 4 Risk-taking and the selection quota

In this section, we consider how contestant risk-taking affects a meritocratic designer’s choice of the selection quota, $m$. For this analysis, we fix contest size, $n$, and apply the same meritocratic designer welfare function adopted in Section 2 to more general contexts—we assume that the designer sets the quota, $m$, to maximize his welfare, $u$, where

\[ u = \mathbb{E}[\text{#Strong selected contestants} - \text{#Weak selected contestants}]. \]

(19)

This welfare function is symmetric—the designer gains one utile by selecting a strong contestant and loses one utile by selecting a weak contestant. In Section 6.1, we show that extending the analysis by introducing asymmetry between gains and losses does not result in any qualitative change of our results.

Note that, when $n$ is fixed, the expected number of weak contestants is fixed, implying that the sum of the expected number of weak selected contestants and the expected number of weak deselected contestants is fixed. Inspection of (19) then shows that, with fixed contest size, $n$,
we can also interpret the designer’s problem as a task-assignment problem—the designer sets $m$ to maximize the expectation of

$$\#\text{Strong selected contestants} + \#\text{Weak deselected contestants}.$$  

In this task-assignment problem, the designer assigns a fixed pool of contestants either to a more desirable “selection task” or a less desirable “deselection task.” The marginal product of strong (weak) contestants is higher when performing the selection (deselection) task. The designer’s problem is to fix the performance rank required for “promotion” to more desirable task.

**Non-strategic contestants** When contestants are non-strategic, weak contestants, because of their lower capacity, never best strong contestants. Hence, strong contestants are prioritized for selection. As discussed in Section 2, the number of strong contestants is Binomially distributed with parameters $n$ and $\theta$, and when contestants are non-strategic, the symmetry of the designer’s welfare function (equation (19)) implies that it is optimal to set the quota, $m$, equal to a median of the Binomial$(n, \theta)$ distribution.

Binomial distributions have either one or two medians. Whenever a Binomial distribution has two medians, the two medians differ by one and are both optimal when contestants are non-strategic.\textsuperscript{21} For expositional convenience, in the subsequent analysis, we assume that, whenever the designer is indifferent between two quotas, he chooses the larger quota.\textsuperscript{22} Under this assumption, the quota selected by the designer when contestants are non-strategic, denoted by $m^*_{M}$, equals the median number of strong contestants, when the median is unique, and equals the larger median, when there are two medians, i.e.,

$$m^*_{M}(n, \theta) = \min\{m \in \{0, 1, \ldots, n\} : B(m; n, \theta) > 1/2\}, \quad (20)$$

where $B(\cdot; n, \theta)$ denotes the CDF of the Binomial$(n, \theta)$ distribution.

If $m^*_{M}$ equals 0 ($n$), then the optimal quota selects none (all) of the contestants even in the absence of risk-taking. In this case, the contest mechanism, which requires the quota to be strictly between 0 and $n$, does not further the goal of meritocratic selection. The examination of the effects of risk-taking on contests when contests cannot further meritocracy is not a very interesting exercise. Thus, in the subsequent analysis, we impose the following restriction:

**Assumption 1.** The optimal quota when contestants are non-strategic is interior, i.e., $0 < m^*_{M} < n$.

**Strategic contestants** When contestants are strategic, changing the quota changes contestant behavior. As the following theorem implies, increasing competition, by setting a smaller quota,

\textsuperscript{21}When the Binomial$(n, \theta)$ distribution has two medians and contestants are non-strategic, if the quota equals the smaller median, the marginal contestant deselected has a probability of being strong equals exactly one half. In this case, when contestants are non-strategic and the designer’s welfare function is symmetric, setting the quota to the larger median is equally optimal as setting the quota to the smaller median.

\textsuperscript{22}All of our results hold if we instead assume that, whenever the designer is indifferent between two quotas, he chooses the smaller quota.
increases weak contestants’ tendency to challenge strong contestants and, consequently, depending on the prior quality of contestants, a meritocratic designer either tends to accommodate risk-taking by “inflating” the quota or shuts down competition by not running a contest.

**Theorem 1.** Suppose contest size, $n$, is fixed but the designer can vary the selection quota, $m$. Let $m^*$ be the optimal quota when contestants are strategic. The optimal quota when contestants are non-strategic, $m^*_M$, is given by equation (20).

i. For $m', m'' \in \{1, 2, \ldots, n - 1\}$, if a challenge (concession) equilibrium exists when $m = m''$ and $m' < (>) m''$, a challenge (concession) equilibrium exists when $m = m'$.

ii. If
\[
\frac{1 - \theta}{\theta} \leq \frac{\mu_S}{\mu_W},
\]
then, (a) if a concession equilibrium exists when $m = m^*_M$, $m^*_M = m^*_M$; (b) if a challenge equilibrium exists when $m = m^*_M$, $m^* \geq m^*_M$ and $m^* = \bar{m}$ or $\bar{m} + 1$, where $\bar{m}$ is the largest quota at which a challenge equilibrium exists, i.e.,
\[
\bar{m} = \max \left\{ m \in \{m^*_M, \ldots, n - 1\} : p^C_{o}(m) < p^G_{o}(m) \right\},
\]
and $p^C_{o}$ and $p^G_{o}$ are given by (6) and (10) respectively.

iii. If condition (21) is violated, a challenge equilibrium exists when $m = m^*_M$, and $m^* = 0$.

The logic behind part (i) of the theorem is fairly straightforward: reducing the quota makes it less likely that besting only weak rivals is sufficient for a weak contestant to be selected. Thus, reducing the quota inclines weak contestants to adopt high-risk strategies that challenge strong contestants.

Part (ii) shows that, when the prior odds that a given contestant is weak, $(1 - \theta)/\theta$, are low relative to the asymmetry in contestant capacity, measured by the capacity ratio, $\mu_S/\mu_W$, a meritocratic designer tends to inflate the quota, setting quotas greater than the optimal quota in the absence of risk-taking. Quota inflation is driven by two factors. First, quota inflation mollifies weak contestants’ risk-taking incentives, making performance a better signal of ability. Second, risk-taking reduces the correlation between ability and contest performance, thereby reducing the quality of top performers and increasing the quality of mediocre performers. When the prior odds of a contestant being weak are fairly small, this effect encourages the designer to dip deeper into the contestant pool by inflating the quota.

In the case described by part (iii), the prior odds of a contestant being weak are large relative to the capacity asymmetry between strong and weak contestants. Low capacity asymmetry makes it easier for weak contestants to challenge strong contestants through risk-taking. Risk-taking, given the high prior odds of a contestant being weak, makes even top performers unworthy of selection. Thus, it is optimal to set a zero quota, or equivalently not conduct a selection contest.

Note that, when contestants are non-strategic, setting a zero quota is never optimal if contest size, $n$, is sufficiently large. In contrast, the condition for the optimality of a zero quota when
contestants are strategic (i.e., $(1-\theta)/\theta \geq \mu_S/\mu_W$) is independent of $n$. Thus, part (iii) implies that risk-taking blocks using highly selective contests to pluck a few high-ability candidates out of a large but weak candidate pool.

5 Risk-taking and contest size

In this section, we consider the effect on designer welfare of varying contest size, $n$, given a fixed selection quota, $m$. The number of strong selected contestants and weak selected contestants sums to $m$, which is fixed by the analysis in this section. Inspection of equation (19) shows that, when $m$ is fixed, the designer’s problem is equivalent to maximizing the expected number of strong selected contestants, i.e., the quality of the $m$ selected contestants.

Non-strategic contestants The effect of varying $n$ when contestants are non-strategic is easy to identify. Given that contestants are non-strategic, strong contestants are prioritized for selection. Suppose we add a new contestant to the contestant pool. If the added contestant is strong, and if before the contestant’s addition, less than $m$ contestants were strong, the new contestant will be selected, and the number of strong selected contestants will increase. Otherwise, i.e., if the new contestant is weak or the selection quota has already been filled by strong contestants, the number of strong selected contestants will not change. Because the ability of each contestant is drawn independently, the probability that the pool contains less than $m$ strong contestants is always positive. Thus, when the quota is fixed and contestants are non-strategic, adding contestants increases the expected number of strong selected contestants and thus designer welfare.

Strategic contestants Now consider the effect of varying $n$ when contestants are strategic. As our next theorem shows, increasing $n$ through pool expansion inclines weak contestants to adopt high-risk strategies that challenge strong contestants. Consequently, while pool expansion increases the expected number of strong candidates in the pool, because of increased risk-taking, the increased expected number of strong candidates need not translate into an increased expected number of strong selected contestants. In fact, as our theorem implies, when contest size is sufficiently large, further increasing size does not change the expected number of strong selected contestants.

Theorem 2. Suppose the selection quota, $m$, is fixed but the designer can vary contest size, $n$.

i. For $n', n'' > m$, if a challenge (concession) equilibrium exists when $n = n''$ and $n' > (<) n''$, a challenge (concession) equilibrium also exists when $n = n'$.

ii. There exists $n^c$ such that a challenge equilibrium exists for all $n > n^c$.

iii. If a challenge equilibrium exists at $n = n'$, designer welfare at any $n > n'$ equals designer welfare at $n = n'$.

The basic implication of Theorem 2 is that risk-taking caps the gains from inclusivity. When making the contestant pool more inclusive is costly because of outreach, advertisement, or
search costs, the optimal contest size when contestants are strategic risk-takers will tend to be smaller than when contestants are non-strategic. As we will show in Online Appendix C, even when increasing contest size is costless, when the pool of potential new contestants is, on average, of lower quality than the incumbent candidate pool, the designer may strictly gain from excluding the potential contestants from the contest. In such cases, the gain from expanding the pool produced by increasing the expected number of strong candidates is overwhelmed by the cost of increased risk-taking. In contrast, when contestants are non-strategic, the designer always strictly gains from inclusion because adding contestants increases the expected number of strong candidates.

Thus, when contestants are strategic risk-takers, even meritocratic designers, who are not biased toward specific candidates, have little incentive to expand candidate fields and sometimes will deliberately restrict consideration to candidates who, ex-ante, look promising, even if considering a wider field is costless. Our analysis also implies that the optimality of opening an in-house competition to external candidates depends on whether the in-house competition is soft and whether the external candidates are expected to be stronger than internal candidates, but does not depend on the number of potential external candidates.

6 Extensions and applications

In this section, we consider various modifications of our baseline model. These extensions show that our results are quite robust and also lead to new implications.

6.1 Asymmetric designer objective

In Sections 4 and 5, we investigated the effect of risk-taking on the design of selection contests when a meritocratic designer has a symmetric welfare function given by (19). In what follows, we show that the qualitative nature of our analysis extends to the case where designer welfare, $u$, has the following asymmetric specification:

$$u = \mathbb{E}[(1 - \sigma) \times \#\text{Strong selected contestants} - \sigma \times \#\text{Weak selected contestants}], \quad (22)$$

where $0 < \sigma < 1$.

Allowing for asymmetry between gains from selecting strong contestants and losses from selecting weak contestants does not change any of our result in Section 5, where the designer can only vary contest size, $n$, but not the quota, $m$. This is because the number of weak selected contestants equals the quota, $m$, less the number of strong selected contestants, implying that the designer’s problems under the two objective functions, (19) and (22), are equivalent for any fixed $m$.

Now consider how introducing asymmetry into the designer’s objective function might affect our analysis made in Section 4, where the designer can only vary the quota, $m$, but not contest size $n$. As our next proposition shows, the qualitative conclusions of Theorem 1 de-
rived under the symmetric objective specification, (19), extend to the asymmetric objective specification, (22).

**Proposition 1.** When designer objective is given by the asymmetric specification (22), then Theorem 1 holds under two modifications: first, modify the expression for the optimal quota in the case of non-strategic contestants, $m^*_M$, from equation (20) into

$$m^*_M(n, \theta, \sigma) = \min\{m \in \{0, 1, \ldots, n\} : B(m; n, \theta) > 1 - \sigma\}. \quad (23)$$

Second, modify condition (21) into

$$\frac{1 - \theta}{\theta} \leq \frac{\mu_S}{\mu_W} \left(1 - \frac{\sigma}{\sigma}\right). \quad (24)$$

Proposition 1 implies that, under both the symmetric, equation (19), and asymmetric specification, equation (22), of the designer’s objective function, the designer tends to inflate the quota if the prior odds of a contestant being weak are sufficiently low relative to the asymmetry in contestant capacity, measured by $\mu_S/\mu_W$, and sets a zero quota if otherwise.

**Application: Retention contests**

Although Proposition 1 shows that asymmetric specifications of designer welfare do not produce any qualitative change in the conclusions of our analysis, the asymmetric specification of welfare considered in Proposition 1 does extend the scope of application of our analysis to settings where there are strong reasons to suspect that the designer’s welfare function is asymmetric.

One such setting is retention contests. In a retention contest, the designer sets a “retention quota,” $m$, the number of incumbent workers who will be retained. The remaining workers will be dismissed. The retention quota is filled by the incumbent workers with the best relative performance rankings. Dismissed workers will be replaced by an equal number of new hires.

In practice, retention contests are fairly common and are frequently engendered by “forced-ranking systems.” Forced-ranking systems, used by an estimated 20% of large U.S. corporations (Bates, 2003), assign a fixed percentage of employees (within a given comparison group) performance grades based on relative performance. The performance of employees assigned the lowest grade is deemed to be below the retention bar. In this setting, the retention quota corresponds to the number of employees in the $n$-employee comparison group who will not receive the lowest performance grade.

We consider a retention contest in which a firm sets a retention quota for $n \geq 2$ incumbent workers, each of whom is strong with probability $\theta$ and weak with probability $1 - \theta$. A worker’s type is the worker’s private information. The firm sets a retention quota, $m$. Under the quota, $m$ workers will be retained, the $n - m$ bottom performers will be dismissed, and the firm will recruit $n - m$ workers to fill the vacancies produced by dismissal. The firm knows that each new hire will be strong with probability $\sigma$ and weak with probability $1 - \sigma$. Given that, in practice, incumbent workers have retained their positions over some time period during
which their performance has been observed, to some degree, incumbent workers have been better screened for quality than new hires. It thus seems natural to assume that the expected quality of incumbent workers is no less than the expected quality of new hires.

Assumption 2. In retention contests, $\theta \geq \sigma$.

The firm’s problem is to set the retention quota, $m$, to optimize workforce quality, i.e., to maximize

$$E[\#\text{Strong retained workers} + \#\text{Strong new hires}].$$  \hfill (25)

Comparing the asymmetric designer objective function, given by equation (22), and the retention contest objective function, equation (25), might give the impression that these objective functions are quite distinct. In fact, they are equivalent. To see this, first note that, in equation (25), the expected number of strong new hires equals the number of new hires, $n - m$, multiplied by the probability that a new hire is strong, $\sigma$. Thus, given that $n$ and $\sigma$ are both fixed, maximizing the objective in (25) is equivalent to maximizing

$$E[\#\text{Strong retained workers}] - \sigma m.$$  

Because $m$ workers are retained, $m = E[\#\text{Strong retained workers}] + E[\#\text{Weak retained workers}]$. Thus,

$$E[\#\text{Strong retained workers}] - \sigma m = E[(1 - \sigma) \times \#\text{Strong retained workers} - \sigma \times \#\text{Weak retained workers}].$$  \hfill (26)

Comparing the retention contest objective function, (26), and the asymmetric designer objective function provided by (22), shows that these objective functions are equivalent.

Given this equivalence, Proposition 1 can be used to produce a characterization of the effect of strategic risk-taking on retention quotas. Note that the condition in Proposition 1 for selection to be biased toward quota inflation (condition (24)) is always satisfied for retention contests under Assumption 2.

Result 1. In retention contests,

(i) the optimal retention quota in the risk-taking contest always weakly and sometimes strictly exceeds the optimal quota when contestants are non-strategic.

(ii) Absent strategic risk-taking, the optimal quota given contest size $n$, $m^*_M(n)$, satisfies

$$\lim_{n \to \infty} m^*_M(n)/n = \theta.$$  

The optimal quota in the risk-taking contest given contest size $n$, $m^*(n)$, satisfies, $\lim_{n \to \infty} m^*(n)/n = \theta + (1 - \theta)(\mu_W/\mu_S) > \theta$.

Part (i) of Result 1 implies that firms tend to “over-retain” their employees, setting retention rates greater than what would be optimal if employees were non-strategic. Moreover, as part (ii) shows, if the retention contest is applied to a sufficiently large group, the firm will always inflate the retention rate regardless of the degree of capacity asymmetry. The degree of inflation equals

$$(1 - \theta)(\mu_W/\mu_S),$$  

which is proportional to the fraction of incumbent workers who are weak and
inversely proportional to the capacity ratio, $\mu_S/\mu_W$. Thus, when the worker group is sufficiently large, high ability is rare, and contest performance is positively but not that strongly related to ability, strategic risk-taking will substantially increase the retention quota. For example, if the capacity ratio is $3:2$ and the prior probability that a given worker is strong equals $1/4$, asymptotically, 25% of incumbents will be retained when incumbents are non-strategic, while 75% will be retained when incumbents are strategic.

### 6.2 Capacity acquisition

In our baseline model, we assumed that a contestant’s capacity to perform in the contest, which fixes the contestant’s expected performance, is exogenous. In this subsection, we allow each contestant to acquire capacity through costly effort, e.g., a fund manager, by investing effort in gathering and analyzing information, can better identify undervalued stocks and thus improve expected portfolio performance. We assume that, after the selection quota and contest size are announced to the contestants, the contestants first simultaneously choose capacity by exerting costly effort. The effort cost function is a strictly convex power function: the cost of choosing the capacity level $\mu$ for a type-$t \in \{S, W\}$ contestant is $c_t(\mu) = \mu^\alpha/a_t$, where $\alpha > 1$ and $a_t$ is an ability parameter satisfying $0 < a_W < a_S$. Thus, the cost and the marginal cost of acquiring any given capacity level are both higher for weak contestants than for strong contestants. After the contestants acquire their capacity, the contestants, without knowing each other’s capacity, simultaneously choose nonnegative random performance subject to their capacity constraints. Selection is still based on the ranking of realized performance. A contestant’s reward from being selected is greater than her reward from being deselected. A contestant’s payoff equals the reward she receives less her effort cost.

As our next result implies, introducing capacity acquisition into our baseline model endogenizes the capacity ratio, $\mu_S/\mu_W$, without any other change of our analysis.

**Proposition 2.** The modified game in which capacity is acquired through costly effort produces the same selection outcomes, i.e., each type’s probability of winning and meritocratic designer welfare (22), as the baseline model with $\mu_S/\mu_W = (a_S/a_W)^{1/(\alpha-1)}$.

Proposition 2 implies that, to analyze the selection outcomes of the modified game, we can simply examine the selection outcomes of our baseline model, treating contestants as if their capacity ratio, $\mu_S/\mu_W$, equaled the value of $(a_S/a_W)^{1/(\alpha-1)}$. Because this value does not depend on the selection quota, $m$, or contest size, $n$, all of our previous analysis of how risk-taking affects the design of selection contests is robust to our extension here.

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23 By Lemma 1, a weak contestant’s probability of winning equals max $[p^C_o, p^G_o]$, where $p^C_o$ does not depend on $\mu_S$ or $\mu_W$ while $p^G_o$ depends on $\mu_S/\mu_W$ but not on $\mu_S$ or $\mu_W$ per se. Thus, as long as $\mu_S/\mu_W$ is fixed, selection outcomes are independent of the absolute values of $\mu_S$ and $\mu_W$. 

22
Application: Introducing effort into designer welfare

The central conclusion of our analysis is that, when contestants are strategic, even purely meritocratic contest designers are biased toward implementing “uncompetitive,” contest designs that consider limited talent pools and offer places to contestants who are expected to be unworthy of selection. The natural approach to establishing this conclusion, and the one we have taken in this paper, is to model a contest designer who is only concerned with the selection effects of contest design.

Although this formulation allows for a clean identification of the effects of meritocratic selection preferences on contest design, it does raise questions about the relevance of our insights in cases where contestant performance affects designer welfare through alternative channels. The most obvious alternative channel is the effort channel, i.e., when the effort applied by some contestants produces valuable outputs owned by the designer.

For example, in job-assignment and retention contests, “internal contestants,” currently-employed workers, exert effort that produces goods and services. In CEO promotion contests, internal contestants, the rival incumbent managers, make management decisions that affect firm value. In such contests, both the effort of internal contestants and the quality of selected (internal or external) contestants will affect designer welfare.

If the effort-based welfare effects of quota inflation and pool restriction were opposed to the selection-based effects, then the predictions of our model for such contests would depend on the balance between the magnitudes of these effects.

However, in what follows, we show, using the endogenous-capacity framework developed in this section, that the effort-based welfare effects never oppose and sometimes amplify the incentive to inflate quotas and restrict contest participation. Following the approach used in much of the effort-based contest-design literature, we compare different contest designs that feature the same total reward budget, $V > 0$. These designs will assign a reward to each contestant that is conditioned on whether the contestant is selected. Because, ceteris paribus, decreasing the rewards for deselected contestants will increase effort without any effect on meritocratic welfare, it is always optimal to assign zero rewards to deselected contestants. Thus, given contest budget $V$ and selection quota $m$, each selected contestant will receive a reward of $V/m$ and each deselected contestant will receive a reward of 0.

Now consider effort in such contests. Let $\bar{\mu}$ represent expected individual contestant effort. Because, in any symmetric equilibrium, contestants of type $t$, $t \in \{S, W\}$, all choose the same effort (capacity), denoted by $\mu_t$, and because, ex-ante, each contestant is strong with probability $\theta$ and weak with probability $1 - \theta$,

$$\bar{\mu} = \theta \mu_S + (1 - \theta) \mu_W.$$ 

Our next result, the key to establishing our conclusions, describes the effort effects of changes in the selection quota and contest size in challenge equilibria.
Result 2. Given a fixed total reward budget, $V$, (i) for fixed contest size, $n$, all selection quotas, $0 < m < n$, that support challenge equilibria produce the same expected individual contestant effort, $\bar{\mu}$; (ii) for a fixed selection quota, $m$, and all $n', n'' > m$, if a challenge equilibrium exists at $n = n'$ and $n'' > n'$, expected individual contestant effort, $\bar{\mu}$, is lower at $n = n''$ than at $n = n'$.

We assume that designer welfare, has two components, a meritocratic component given by $u$ as defined by equation (22) and an effort component. The effort component is a function of the total expected effort of the “internal contestants” in the contest. A contestant is internal if the output produced by the contestant’s effort is owned by the designer. When the number of internal contestants is fixed, the total expected effort of the internal contestants is determined by expected individual contestant effort, $\bar{\mu}$. We assume that designer welfare is strictly increasing in both the effort component and meritocratic component.

First, consider quota selection when all contestants are internal and contest size is fixed, for example, retention and job-assignment contests. In the endogenous-capacity framework, Propositions 1 and 2 imply that, if the designer’s welfare function were purely meritocratic, the optimal selection quota would be greater than the optimal quota in the absence of risk-taking whenever the optimal selection quota is positive (i.e., condition (24) is satisfied) and the optimal selection quota in the absence of risk-taking is less than the largest quota that supports challenge equilibria.

Now consider, under the same hypotheses, the effect of the effort component on the optimal quota. First note that a zero quota cannot induce any effort and, under the zero quota, the meritocratic component of welfare is also 0. Thus, because condition (24) is satisfied, a zero quota cannot be optimal. Now consider any positive quota less than the highest quota that supports challenge equilibria. Lowering the quota encourages weak contestants to challenge strong contestants (Theorem 1.i); thus such quotas also support challenge equilibria. By Result 2(i), expected individual effort under these quotas will be the same as under the highest quota that supports challenge equilibria.

Therefore, the choice between quotas that support challenge equilibria will be determined by the meritocratic component of the designer’s welfare function. For exactly the same reasons as advanced in Theorem 1, among quotas that support challenge equilibria, the optimal quota is the highest quota that supports a challenge equilibrium. Thus, the optimal quota is at least equal to the largest quota that supports a challenge equilibrium. Consequently, as in our analysis of a purely meritocratic designer, when the designer’s welfare function includes an effort component, the optimal quota will exceed the optimal quota in the absence of risk-taking.

Next, consider optimal contest size when the selection quota is fixed, e.g., a CEO selection contest. Suppose the designer’s problem is whether to run an “in-house” competition restricted to internal candidates or open up the contest to external candidates.

If the designer’s welfare function were purely meritocratic, Proposition 2 and Theorem 2 would imply that, when the number of internal contestants is large enough to ensure that the in-
house competition supports a challenge equilibrium, opening the contest to external candidates would not affect designer welfare.

Now consider, under the same hypotheses, the effect of the effort component on the optimality of in-house competitions. By Result 2(ii), for a fixed quota, when the number of internal contestants is sufficiently large, opening the competition to external candidates reduces the expected individual effort of internal contestants. Because the number of internal contestants is fixed, the effort component depends on expected individual contestant effort. Thus, when the designer’s welfare function has an effort component, the designer strictly prefers excluding external candidates. Including an effort component into the designer’s welfare function strengthens the designer’s preference for in-house competitions. Thus, introducing effort into the designer’s welfare function, in settings where effort concerns are natural, e.g., promotion, retention, and job-assignment contests, does not reverse the basic conclusions of our analysis.

6.3 Upper bounds on performance

Many real-life contests naturally feature an upper bound on contest performance, such as full marks in examinations. If $\bar{x}$ represents the upper bound, contestants are restricted to performance distributions that place no weight above $\bar{x}$. Thus, a performance level equal to $\bar{x}$ cannot be topped. Consequently, in equilibrium, point mass on $\bar{x}$ can possibly exist (cf. the example in Section 2). While imposing an upper bound, $\bar{x}$, on performance can affect contestants’ equilibrium performance distributions, as our next result shows, imposing the upper bound, $\bar{x}$, does not change any equilibrium payoff characterization presented in Lemma 1, provided that $\bar{x} \geq \mu_S$.

**Proposition 3.** Imposing an upper bound, $\bar{x}$, on performance, $\bar{x} \geq \mu_S$, does not change any result in Lemma 1.

Because a meritocratic designer’s welfare hinges on contestants’ payoffs (probabilities of winning) not on their strategies per se, and because, as shown by Proposition 3, imposing an upper bound, $\bar{x}$, on performance, $\bar{x} \geq \mu_S$, does not change equilibrium payoff characterizations, our contest-design analysis is robust to imposing the upper bound, $\bar{x} \geq \mu_S$.

If the upper bound is strictly less than strong contestants’ capacity, i.e., if $\bar{x} < \mu_S$, there is no feasible performance distribution that satisfies strong contestants’ capacity constraint, which requires a contestant’s expected performance to equal her capacity. However, if we relax the capacity constraint by allowing contestants to “burn” capacity in the sense that a contestant’s expected performance need not equal but only be no greater than her capacity, our contest-design analysis is also robust to imposing the upper bound, $\bar{x} \in [\mu_W, \mu_S)$. To see this, note that, if $\bar{x} < \mu_S$, then submitting a fixed performance level equal to the upper bound, $\bar{x}$, is a

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24 The sum of internal and external contestants’ expected effort nevertheless increases.

25 The case in which the upper bound is no greater than weak contestants’ capacity is trivial, because, under such a highly restrictive upper bound, each contestant’s performance will equal the upper bound.

25
dominant strategy for strong contestants. Thus, by Proposition 3, if $\mu_W \leq \bar{\mu} < \mu_S$, in which case only weak but not strong contestants can fully utilize their capacity, the contest with the upper bound $\bar{\mu} \in [\mu_W, \mu_S)$ produces the same selection outcomes as our baseline contest with $\mu_S = \bar{\mu}$. Hence, even if $\bar{\mu} \in [\mu_W, \mu_S)$, as long as the upper bound, $\bar{\mu}$, is fixed, our previous analysis of how risk-taking affects contest design still holds.

**Application: Elite-university admissions**

Elite-university admissions systems are extremely competitive. Because students’ performance assessment is frequently based on their exam outcomes, which naturally have full marks as an upper bound, the fact that our contest-design analysis is robust to imposing an upper bound on performance permits us to draw an implication related to the “relaxed” elite-university admissions system proposed by Schwartz (2007).

Schwartz (2007) argues that, because student performance can be affected by luck, in elite-university admissions competitions, the expected-quality difference between the best-performing students and the close-to-best performing students is likely to be quite small. At the same time, extremely competitive admissions systems impose large psychological costs on students. Thus, the benefits of extremely competitive admissions systems, in terms of slightly more meritocratic selection, are likely to be outweighed by their psychological costs. Hence, elite universities should make their admissions less selective by adopting a “relaxed” admissions system which “approves” more applicants than can be admitted, and fills the admission quota with approved students using a random lottery.

In Schwartz (2007), the random component in student performance is exogenous. Our analysis shows that an even stronger case for Schwartz’s relaxed admissions system can be made when the random component is the product of strategic risk-taking—adopting a relaxed admissions system can reduce the psychological costs of admissions competitions without sacrificing meritocracy.\(^{26}\)

**Result 3.** Suppose that a risk-taking contest with $n$ contestants and $m$ places supports a challenge equilibrium. Then using a “relaxed” selection policy by first approving $m' > m$ contestants based on performance ranks and then randomly selecting $m$ out of these $m'$ approved contestants does not affect designer welfare (defined in equation (22)), provided that the $n$-contestant/$m'$-winner risk-taking contest also supports a challenge equilibrium.

\(^{26}\)Using a large all-pay contest setting, which focuses on contestants’ effort strategies, while abstracting from risk-taking and selection, Olszewski and Siegel (2018) show that making contests less competitive by pooling intervals of performance rankings can improve student welfare in a Pareto sense via reduced student effort, even though pooling reduces the correlation between selection and ability. Our result implies that, if students strategically take risks, reducing competition need not reduce the correlation between selection and ability.


7 Conclusion

In this paper, we studied selection contests designed to implement meritocracy, i.e., select, based on contest performance, strong, more able, contestants and deselect weak, less able, contestants. In contrast to much of the literature on contests and tournaments, which assumes that contest noise is the product of an exogenous “noise term” mediating the relationship between contestant actions and contest performance, in our analysis, “contest noise” is endogenously produced by strategic contestant risk-taking.

Introducing strategic risk-taking has fundamental effects on optimal selection-contest design: increasing competition, either by reducing the number of contestants selected or expanding the contestant pool, increases weak contestants’ tendency to play high-risk strategies that challenge stronger, more able, contestants. Because of this effect, even meritocratic designers can gain by limiting competition through adopting “clubby” contest designs, designs that feature less inclusive contestant pools and relaxed selection standards. Our model implies that many seemingly unmeritocratic practices and proposals—e.g., “Peter-Principle” promotion and retention policies, “in-house” job competitions, and elite-university admissions systems that incorporate a lottery component (Schwartz, 2007)—can, in fact, be consistent with, and sometimes even further meritocracy.

References


Online Appendix to “Less competition, more meritocracy?”

A. Proofs of results

Proof of Lemma 1. In the main text, we proved Lemma 1 based on assuming that (i) Claim 1 holds and (ii) a symmetric equilibrium always exists. To complete the proof, it suffices to establish (i) and (ii). In what follows, we establish (i). We defer the establishment of (ii) to Appendix B, where we construct a symmetric equilibrium.

No point mass on zero performance  We first show, by way of contradiction, that in any symmetric equilibrium, no contestant places point mass on 0. Suppose, to the contrary, that in a symmetric equilibrium, there exists at least one type, say \( t \), such that each type-\( t \) contestant places point mass on 0. Let \( dF_t \) be the performance measure associated with the function of the performance distribution, \( F_t \). Let \( q_t \) be the probability that a given type-\( t \) contestant chooses zero performance, i.e.,

\[
q_t = dF_t(\{0\}).
\]

The hypothesis that each type-\( t \) contestant places point mass on 0 implies that

\[
q_t \in (0, 1).
\] (A-1)

Note that we can decompose \( dF_t \) as follows:

\[
dF_t = q_t \mathbb{1}_0 + (1 - q_t) dF_t^{-0},
\] (A-2)

where

\[
\mathbb{1}_a(x) = \begin{cases} 1 & \text{if } x = a \\ 0 & \text{otherwise} \end{cases}
\] (A-3)

is an indicator function and

\[
dF_t^{-0}(x) = \begin{cases} 0 & \text{if } x = 0 \\ \left( \frac{1}{1 - q_t} \right) dF_t(x) & \text{otherwise} \end{cases}.
\] (A-4)

This decomposition simply means that we can interpret a type-\( t \) contestant’s strategy as choosing zero performance with probability \( q_t \) and choosing a random, strictly positive performance level whose associated probability measure equals \( dF_t^{-0} \) with probability \( 1 - q_t \).

Now consider a type-\( t \) contestant’s deviation to using an alternative performance measure \( d\hat{F}_t \) given as follows:

\[
d\hat{F}_t = \hat{q}_t \mathbb{1}_\varepsilon + (1 - \hat{q}_t) dF_t^{-0},
\] (A-5)

where \( \varepsilon \in (0, \mu_t) \), \( \mathbb{1}_\varepsilon \) and \( dF_t^{-0} \) are given by (A-3) and (A-4), respectively, and

\[
\hat{q}_t = \frac{\mu_t/(1 - q_t) - \mu_t}{\mu_t/(1 - q_t) - \varepsilon}.
\] (A-6)
The hypothesis that \( \varepsilon < \mu_t \) and equation (A-1) imply that \( \hat{q}_t \in (0, 1) \). Thus, by construction and the fact that \( d\hat{F}_t \) is a probability measure, \( d\hat{F}_t \) is also a probability measure. Note that

\[
\int_{0^-}^{\infty} x d\hat{F}_t(x) = \hat{q}_t \varepsilon + (1 - \hat{q}_t) \int_{0^-}^{\infty} x d\hat{F}_t^{-0}(x) = \hat{q}_t \varepsilon + (1 - \hat{q}_t) \int_{0^-}^{\infty} \frac{x}{1 - \hat{q}_t} dF_t(x)
\]

\[
= \hat{q}_t \varepsilon + (1 - \hat{q}_t) \left( \frac{\mu_t}{1 - \hat{q}_t} \right) = \mu_t,
\]

where, in the first line, the first equality follows from equations (A-3) and (A-5) and the second equality from (A-4), and in the second line, the first equality follows from the fact that the mean of \( F_t \) equals \( \mu_t \) and the second equality from (A-6). Thus, by construction, \( d\hat{F}_t \) satisfies the capacity constraint for a type-\( t \) contestant.

Now we show that a type-\( t \) contestant is strictly better off deviating from \( dF_t \) to \( d\hat{F}_t \) for \( \varepsilon > 0 \) sufficiently small. Note that

\[
\lim_{\varepsilon \downarrow 0} \int_{0^-}^{\infty} P(x) d\hat{F}_t(x) = \lim_{\varepsilon \downarrow 0} \hat{q}_t P(\varepsilon) + \lim_{\varepsilon \downarrow 0} (1 - \hat{q}_t) \int_{0^-}^{\infty} P(x) d\hat{F}_t^{-0}(x)
\]

\[
= q_t P(0+) + (1 - q_t) \int_{0^-}^{\infty} P(x) dF_t^{-0}(x) > q_t P(0) + (1 - q_t) \int_{0^-}^{\infty} P(x) dF_t^{-0}(x) = \int_{0^-}^{\infty} P(x) dF_t(x),
\]

(A-7)

where the first line follows from (A-5) and, in the second line, the first equality follows from the fact that, given (A-6), \( \lim_{\varepsilon \downarrow 0} \hat{q}_t = q_t \), and the last equality follows from (A-2). To understand the inequality in the last line, note that, by hypothesis, each type-\( t \) contestant places point mass on 0. Thus, for a given contestant, there exists a positive probability that all her rivals are of type \( t \) and have zero performance. Consequently, if this contestant’s performance equals 0, there exists a positive probability that she will tie with all of her rivals. Hence, given the random resolution of a tie, her probability of winning by performing slightly better than 0 is strictly higher than her probability of winning by having zero performance, i.e.,

\[
P(0+) > P(0).
\]

(A-8)

The first and the last expression in (A-7), together with the strict inequality between them, imply that deviating from \( dF_t \) to \( d\hat{F}_t \) for \( \varepsilon > 0 \) sufficiently small makes a type-\( t \) contestant strictly better off, contradicting that \( dF_t \) is an equilibrium performance measure for type-\( t \) contestants. The result that no contestant places point mass on 0 in any symmetric equilibrium thus follows.

**No point mass on any strictly positive performance level**  We establish this equilibrium property by way of contradiction. Suppose, to the contrary, that in a symmetric equilibrium, there exists at least one type, say \( t \), such that each type-\( t \) contestant chooses a performance level equal to \( a > 0 \) with probability \( \tau > 0 \). Then consider a type-\( t \) contestant deviating to taking a mean-preserving spread of her performance by reducing her probability of choosing \( a \) from \( \tau \) to 0, increasing her probability of choosing 0 by \( z \), and increasing her probability of choosing \( a + \varepsilon \) by \( \tau - z \), where

\[
a \tau = (\tau - z)(a + \varepsilon).
\]

(A-9)
Equation (A-9) implies that the prescribed deviation does not change the contestant’s expected performance. This deviation is thus feasible to the contestant. This deviation increases the contestant’s probability of winning by

\[ \Delta(\varepsilon) = zP(0) + (\tau - z)P(a + \varepsilon) - \tau P(a). \]

By equation (A-9), \( z \) tends to 0 as \( \varepsilon \downarrow 0 \). Thus,

\[ \lim_{\varepsilon \downarrow 0} \Delta(\varepsilon) = \tau (P(a+) - P(a)) > 0, \]

where the inequality follows from the hypotheses that \( \tau > 0 \) and type-\( t \) contestants place mass on \( a \) and an argument similar to the one used for establishing (A-8) (under the hypothesis we made there that type-\( t \) contestants place mass on 0). This inequality implies the existence of a profitable deviation for a type-\( t \) contestant and thus, a contradiction. The result that no contestant places point mass on any \( a > 0 \) in any symmetric equilibrium thus follows.

**P is everywhere continuous.** This follows from the fact that no contestant places point mass.

In the rest of the proof of Claim 1, we will adopt the concept of a fair gamble (between two points) defined in the main text and will use the following result:

**Result A-1.** In any symmetric equilibrium, for each type \( t = S, W \), a fair gamble between \( x' \) and \( x'' \), where \( x', x'' \in \text{Supp} P_t \), for a type-\( t \) contestant must be a best response for a type-\( t \) contestant. Moreover, all pairs \( (x, P(x)) \) such that \( x \in \text{Supp} P_t \) must be collinear.

**Proof.** This result is developed in the main text. See the argument around Figure 1. Its development only relies on the continuity of \( P \), which has been established above. \( \square \)

**The lower bound of the support of \( P \) is zero.** Let \( \underline{x} = \min \text{Supp} P_t \). By equation (15), there exists at least one type, say \( t \), such that \( \min \text{Supp} P_t = \min \text{Supp} P = \underline{x} \). Because no contestant places point mass in any symmetric equilibrium and because the mean of \( F_t \) equals \( \mu_t \), there must exist \( x'' > \mu_t \) such that \( x'' \in \text{Supp} P_t \). Given that \( x'' < \underline{x} \), Result A-1 implies that a fair gamble between \( \underline{x} \) and \( x'' \) for a type-\( t \) contestant is a best response for a type-\( t \) contestant. To obtain a contradiction, suppose, to the contrary, that \( \underline{x} > 0 \). Then, by equation (15) and the fact that no contestant places point mass in any symmetric equilibrium, \( P(0) = P(\underline{x}) = 0 \). Because \( x'' > \mu_t \) and because weak contestants have to place weight over \([0, \mu_W]\) to satisfy their capacity constraint, it must be that \( P(x'') > 0 \). Thus, \( P(x'') > P(\underline{x}) = P(0) \). Hence, if \( \underline{x} > 0 \), for a type-\( t \) contestant, a fair gamble between 0 and \( x'' \) would produce a strictly higher payoff than a fair gamble between \( \underline{x} \) and \( x'' \), contradicting that the latter strategy is a best response for a type-\( t \) contestant. The result that \( \underline{x} = 0 \) thus follows.

**P has a connected support.** We establish this equilibrium property by way of contradiction. Suppose, to the contrary, that \( P \) has a gap in its support. Then there must exist \( 0 \leq x' < x'' \) such that \([x', x''] \cap \text{Supp} P = \{x', x''\}\). Then by equation (15), no contestant places weight over \((x', x'')\). Thus, \( P \) is flat over \((x', x'')\) and, by continuity of \( P \),

\[ P(x) = P(x'') \quad x \in [x', x'']. \]

(A-10)

**A-3**
Because \([x', x''] \cap \text{Supp}_t P = \{x', x''\}\), equation (15) implies that at least one type of contestant, say \(t\), has \(x'' \in \text{Supp}_t\). Because no contestant places point mass in any symmetric equilibrium, given \(x'' \in \text{Supp}_t\) and given \((x', x'') \cap \text{Supp}_t P = \emptyset\) and \(a \text{ fortiiori} (x', x'') \cap \text{Supp}_t = \emptyset\), there must exist \(\epsilon > 0\) such that \([x'', x'' + \epsilon] \in \text{Supp}_t\). Thus, by Result A-1, \(P\) must be affine in \(x\) for \(x \in [x'', x'' + \epsilon]\). Because \([x'', x'' + \epsilon] \in \text{Supp}_t\), \(P\) must be increasing in \(x\) for \(x \in [x'', x'' + \epsilon]\).

Thus, by equation (A-10), \(P\) is constant in \(x\) for \(x \in [x', x'']\) and is increasing and affine in \(x\) for \(x \in [x'', x'' + \epsilon]\), with a kink at \(x''\). Hence, over \(x \in [x', x'' + \epsilon]\), \(P\) is convex and nonlinear. Thus, by Jensen’s inequality, placing weight in the interior of the interval \([x', x'' + \epsilon]\) is strictly dominated by a mean-preserving spread that transfers all the weight from the interior of this interval to \(x'\) and \(x'' + \epsilon\), the two endpoints of this interval. Hence, a type-\(t\) contestant must place no weight over \((x', x'' + \epsilon)\). This however implies, given \(x' < x''\), that a type-\(t\) contestant must place no weight over \((x'', x'' + \epsilon)\), contradicting that \([x'', x'' + \epsilon] \in \text{Supp}_t\). The contradiction implies that \(P\) has a connected support.

**P has a bounded support.** For each type \(t = S, W\), the fact that \(F_t\) is continuous implies the existence of \(x'\) and \(x''\) such that \(x' \neq x''\) and \(x', x'' \in \text{Supp}_t\). Let \(\beta_t\) be the slope of the line connecting the two points, \((x', P(x'))\) and \((x'', P(x''))\). By equation (15), the fact that \(x', x'' \in \text{Supp}_t\) implies that \(x', x'' \in \text{Supp}_P\). Thus, given that \(P\) is continuous and has a connected support and given that \(x' \neq x''\), \(P(x') \neq P(x'')\). Hence, given that \(P\) is nondecreasing, \(\beta_t > 0\).

Note that

\[
\forall x \in \text{Supp}_t, \quad \beta_t (x - x') = P(x) - P(x') \leq 1 - P(x'),
\]

where the equality follows from Result A-1 and the fact that \(x'' \in \text{Supp}_t\), and the inequality follows from the fact that \(P\), being a probability-of-winning function, is bounded above by 1. Given that \(\beta_t > 0\), equation (A-11) implies that

\[
\forall x \in \text{Supp}_t, \quad x \leq x' + (1 - P(x')) / \beta_t,
\]

which further implies that, for each type \(t = S, W\), the support of \(F_t\) is bounded above. Thus, by equation (15), the support of \(P\) is bounded above.

Claim 1 follows immediately from the facts that \(P\) is continuous and has a bounded and connected support, with 0 as the lower bound of the support. \(\Box\)

**Proof of Theorem 1.** (i): The definitions of \(p^C_\theta\) and \(p^G_\theta\), given in (6) and (10), respectively, imply that

\[
\frac{p^C_\theta}{p^G_\theta} = \mathbb{E} \left[ \max \left[ 1 - \frac{\tilde{S}_n}{m}, 0 \right] \right] \left( \frac{\theta \mu_S + (1 - \theta) \mu_W}{(1 - \theta) \mu_W} \right),
\]

where \(\tilde{S}_n \sim \text{Binomial}(n, \theta)\). Note that, for any fixed \(s, m \mapsto \max \left[ 1 - (s/m), 0 \right]\) is nondecreasing. Because a change in \(m\) does not change the distribution of \(\tilde{S}_n, m \mapsto \mathbb{E} \left[ \max \left[ 1 - (\tilde{S}_n/m), 0 \right] \right]\) must be nondecreasing. Hence, by (A-12), an increase in \(m\) weakly increases \(p^C_\theta/p^G_\theta\). Part (i) then follows immediately from Lemma 1.
(ii): Note that we can express designer welfare, \( u \), given by (19) as follows:

\[
\begin{align*}
\text{(A-13)} \\
\end{align*}
\]

\( u(m) = m \left( 1 - \frac{2(1-\theta)p_w}{m/n} \right) = m \left( 1 - \frac{2(1-\theta)}{m/n} \max \left[ p_C, p_G \right] \right) ,
\]

where the first equality follows from (13) and the second from Lemma 1. Define

\[
\begin{align*}
\text{(A-14)} \\
\end{align*}
\]

\( u_C(m) := m \left( 1 - \frac{2(1-\theta)p_C}{m/n} \right) \)

\( u_G(m) := m \left( 1 - \frac{2(1-\theta)p_G}{m/n} \right) .
\]

Note that \( u_C(m) (u_G(m)) \) represents designer welfare when a concession (challenge) equilibrium exists at the quota, \( m \), and by (A-13),

\[
\begin{align*}
\text{(A-16)} \\
\end{align*}
\]

\( u(m) = \min[u_C(m), u_G(m)]. \)

Because designer welfare in a concession equilibrium equals designer welfare when contestants are non-strategic, we can also interpret \( u_C \) as designer welfare when contestants are non-strategic. By definition, \( m^*_M \) is the optimal quota when contestants are non-strategic. Recall that, if there exists more than one quota that maximizes designer welfare, the optimal quota is defined as the largest quota among those that maximize designer welfare. Thus,

\[
\begin{align*}
\text{(A-17)} \\
\end{align*}
\]

\( u_C(m^*_M) \geq u_C(m), \quad \forall m \in \{0, \ldots, n\}, \) with strict inequality for \( m > m^*_M \).

Now consider part (ii)(a). If a concession equilibrium exists when \( m = m^*_M \), then \( u(m^*_M) = u_C(m^*_M) \). In this case,

\[
\begin{align*}
\text{(A-18)} \\
\end{align*}
\]

\( u(m^*_M) = u_C(m^*_M) \geq u_C(m) \geq \min[u_C(m), u_G(m)] = u(m), \quad m = 0, \ldots, n, \)

where the first inequality follows from (A-17) and is strict for \( m > m^*_M \), and the last equality follows from (A-16). Thus, by (A-18), when contestants are strategic, if a concession equilibrium exists when \( m = m^*_M \), \( m^*_M \) maximizes designer welfare and there exists no \( m > m^*_M \) that also maximizes designer welfare. In this case, \( m = m^*_M \) is the optimal quota when contestants are strategic. This proves part (ii)(a).

Next, consider part (ii)(b). Suppose that a challenge equilibrium exists when \( m = m^*_M \) and suppose that condition (21) is satisfied. Given that \( \bar{m} \), by definition, represents the largest quota at which a challenge equilibrium exists, it must be that

\[
\begin{align*}
\text{(A-19)} \\
\end{align*}
\]

\( \bar{m} \geq m^*_M. \)

By part (i), which has been established, a challenge equilibrium exists for all \( 1 \leq m \leq \bar{m} \). Thus, given that \( u(m) = u_G(m) \) if a challenge equilibrium exists at the quota, \( m \), \( u(m) = u_G(m) \) for \( m = 1, \ldots, \bar{m} \). Plug the expression for \( p_o^G \) given in (10) into equation (A-15) and simplify the result. This yields

\[
\begin{align*}
\text{(A-20)} \\
\end{align*}
\]

\( u_G(m) = m \left( \frac{\theta \mu_S - (1-\theta)\mu_W}{\theta \mu_S + (1-\theta)\mu_W} \right). \)

Given the hypothesis that condition (21) is satisfied, \( m \mapsto u_G(m) \) is nondecreasing and positive.
for $m > 0$. Thus, given that $u(m) = u^G(m)$ for $m = 1, \ldots, \bar{m}$ and given the fact that $u(0) = 0$, it must be that
\[ u(\bar{m}) \geq u(m), \quad m = 0, \ldots, \bar{m} - 1. \]  
(A-21)

By the definition of $\bar{m}$,
\[ u(m) = u^C(m), \quad m = \bar{m} + 1, \ldots, n. \]  
(A-22)

Plug the expression for $p^C_i$, given by the last expression in the first line of (6), into (A-14) and simplify the result. This yields
\[ u^C(m) = 2\mathbb{E} [\min[\tilde{S}_n, m]] - m, \]  
(A-23)

where $\tilde{S}_n \sim \text{Binomial}(n, \theta)$. By (A-23),
\[ u^C(m + 1) - u^C(m) = 2 (\mathbb{E} [\min[\tilde{S}_n, m + 1]] - \mathbb{E} [\min[\tilde{S}_n, m]]) - 1. \]  
(A-24)

Because
\[ \mathbb{E} [\min[\tilde{S}_n, m + 1]] = \mathbb{P}[\tilde{S}_n \geq m + 1](m + 1) + \sum_{s=0}^{m} \mathbb{P}[\tilde{S}_n = s]s \]  
(A-25)

\[ \mathbb{E} [\min[\tilde{S}_n, m]] = \mathbb{P}[\tilde{S}_n \geq m + 1]m + \sum_{s=0}^{m} \mathbb{P}[\tilde{S}_n = s]s, \]  
(A-26)

equation (A-24) implies that
\[ u^C(m + 1) - u^C(m) = 2\mathbb{P}[\tilde{S}_n \geq m + 1] - 1 = 2(1 - \mathbb{P}[\tilde{S}_n \leq m]) - 1 = 2 \left( \frac{1}{2} - B(m; n, \theta) \right), \]  
(A-27)

where $B(\cdot; n, \theta)$ denotes the CDF of the Binomial$(n, \theta)$ distribution. By the definition of $m^n_M$ given in equation (20) and by the fact that $m \mapsto B(m; n, \theta)$ is increasing,
\[ B(m; n, \theta) > \frac{1}{2}, \quad m = m^n_M, \ldots, n. \]  
(A-28)

Equations (A-27) and (A-28) imply that
\[ u^C(m + 1) - u^C(m) < 0, \quad m = m^n_M, \ldots, n - 1. \]

Thus, by (A-19) and (A-22),
\[ u(\bar{m} + 1) > u(m), \quad m = \bar{m} + 2, \ldots, n. \]  
(A-29)

Equations (A-21) and (A-29) imply that the optimal quota when contestants are strategic is either $\bar{m}$ or $\bar{m} + 1$.28 This establishes part (ii)(b).

(iii): Suppose that condition (21) is violated. Then, by (A-20), $u^G(m) < 0$ for all $m > 0$. Equation (A-16) implies that $u(m) \leq u^G(m)$ for all $m > 0$. Thus, $u(m) < 0$ for all $m > 0$. Given that $u(0) = 0$, it is optimal to choose $m = 0$. Also note that the fact that $u(m) < 0$ for all $m > 0$ and the hypothesis that $0 < m^n_M < n$ imply that $u(m^n_M) < 0$. Given that $m^n_M$, by definition,

27 If $m = n$, all contestants will be selected. In this case, it is weakly optimal for weak contestants to play concession strategies. Thus, $u(m) = u^C(m)$ when $m = n$.

28 If there exist multiple quotas that maximize designer welfare, the optimal quota is defined as the largest among those quotas.
maximizes designer welfare when contestants are non-strategic, \( u^C(m^*_M) \geq u^C(0) = 0 \). Thus, 
\( u^C(m^*_M) > u(m^*_M) \), which, by (A-14), (A-15), and (A-16), implies that \( p^C_0(m) < p^G_0(m) \) when \( m = m^*_M \). Thus, by Lemma 1, challenge equilibria exist when \( m = m^*_M \). This establishes part (iii) and completes the proof of Theorem 1.

\( \square \)

Proof of Theorem 2. (i): Consider the ratio, \( p^C_0 / p^G_0 \), given by (A-12). Note that \( s \mapsto \max[1 - (s/m), 0] \) is nonincreasing. Also note that, when \( n \) increases, the distribution of \( \bar{S}_n \) after the increase stochastically dominates the one before the increase. Thus, \( n \mapsto \mathbb{E} \left[ \max \left[ 1 - (\bar{S}_n/m), 0 \right] \right] \) is nonincreasing. Thus, by (A-12), an increase in \( n \) weakly decreases \( p^C_0 / p^G_0 \). Part (i) then follows immediately from Lemma 1.

(ii): Note that
\[
\mathbb{E} \left[ \max \left[ 1 - \frac{\bar{S}_n}{m}, 0 \right] \right] = \mathbb{P}[\bar{S}_n < m] \mathbb{E} \left[ 1 - \frac{\bar{S}_n}{m} \right] \mathbb{P}[\bar{S}_n < m] \leq \mathbb{P}[\bar{S}_n < m]. \tag{A-30}
\]
Because \( \bar{S}_n \sim \text{Binomial}(n, \theta) \), \( \mathbb{P}[\bar{S}_n < m] \to 0 \) as \( n \to \infty \). Thus, by (A-12) and (A-30), \( p^C_0 / p^G_0 \to 0 \) as \( n \to \infty \). Part (ii) then follows immediately from Lemma 1.

(iii): Suppose that a challenge equilibrium exists at \( n = n' \). Then by part (i), which has been established, a challenge equilibrium also exists at any \( n > n' \). By equation (A-20), designer welfare in challenge equilibria, \( u^G \), does not depend on \( n \). Thus, designer welfare at any \( n > n' \) equals designer welfare at \( n = n' \). Part (iii) then follows.

\( \square \)

Proof of Proposition 1. Part (i) of Theorem 1 does not depend on how we specify the designer’s objective. Thus, introducing asymmetry into designer objective does not change any conclusion in part (i) of Theorem 1.

Now consider how introducing asymmetry into designer objective changes parts (ii) and (iii) of Theorem 1. Let \( \hat{u} \) be designer welfare when designer objective is given by equation (22). Note that
\[
\hat{u}(m) = (1 - \sigma) n \theta p_S - \sigma n (1 - \theta) p_W = (1 - \sigma) (m - n (1 - \theta) p_W) - \sigma n (1 - \theta) p_W
\]
\[
= m \left( 1 - \sigma - \frac{(1 - \theta) p_W}{m/n} \right), \tag{A-31}
\]
where the second equality follows from equation (12). Define
\[
\hat{u}^C(m) := m \left( 1 - \sigma - \frac{(1 - \theta) p^C_0}{m/n} \right) \tag{A-32}
\]
\[
\hat{u}^G(m) := m \left( 1 - \sigma - \frac{(1 - \theta) p^G_0}{m/n} \right). \tag{A-33}
\]
Note that \( \hat{u}^C(m) (\hat{u}^G(m)) \) represents designer welfare when a concession (challenge) equilibrium exists at the quota, \( m \), and by equation (A-31) and Lemma 1,
\[
\hat{u}(m) = \min[\hat{u}^C(m), \hat{u}^G(m)]. \tag{A-34}
\]
Equation (A-32) and the expression for \( p^C_0 \), given by the last expression in the first line of
equation (6), imply that
\[ \hat{u}^C(m) = \mathbb{E}[\text{min}[\tilde{S}_n, m]] - m\sigma. \]
Thus,
\[ \hat{u}^C(m + 1) - \hat{u}^C(m) = \mathbb{E}[\text{min}[\tilde{S}_n, m + 1]] - \mathbb{E}[\text{min}[\tilde{S}_n, m]] - \sigma = P[\tilde{S}_n \geq m + 1] - \sigma \\
= 1 - P[\tilde{S}_n \leq m] - \sigma = 1 - B(m; n, \theta) - \sigma, \quad (A-35) \]
where \( B(\cdot; n, \theta) \) denotes the CDF of the Binomial(\( n, \theta \)) distribution, and the second equality in the first line follows from (A-25) and (A-26). Equation (A-35) implies that
\[ \hat{u}^C(m + 1) > (=)(<)\hat{u}^C(m) \quad \text{if} \quad B(m; n, \theta) < (=)(>)1 - \sigma. \]
Thus, given that \( m \mapsto B(m; n, \theta) \) is increasing, \( \hat{u}^C \) is first increasing and then decreasing in \( m \), with its maximum reached at \( m = m^*_M \), where \( m^*_M \), whose expression is given by (20) when designer objective has the symmetric specification (19), is now given by (23) when designer objective has the asymmetric specification (22). Because designer welfare when contestants are non-strategic equals designer welfare produced by concession equilibria, \( m^*_M \), given by (23), is the optimal quota in the case of non-strategic contestants.\(^{29} \)

In what follows, let \( m^*_M \) be defined as in equation (23). If a concession equilibrium exists at \( m = m^*_M \), then by the same argument as used in the proof of part (ii)(a) of Theorem 1, \( m^*_M \) is the optimal quota when contestants are strategic. Now consider the case in which a challenge equilibrium exists at \( m = m^*_M \). Note that, by (10) and (A-33),
\[ \hat{u}^G(m) = m \left( 1 - \sigma - \frac{(1 - \theta)\mu_W}{\theta \mu_S + (1 - \theta)\mu_W} \right). \quad (A-36) \]
Thus, if condition (24) is satisfied, \( m \mapsto \hat{u}^G(m) \) is nondecreasing. Then by essentially the same argument as used in the proof of part (ii)(b) of Theorem 1, when a challenge equilibrium exists at \( m = m^*_M \) and when condition (24) is satisfied, the optimal quota given strategic contestants is either \( m \) or \( m + 1 \), where \( m \) represents the largest quota that produces a challenge equilibrium. If condition (24) is violated, then by (A-36), \( \hat{u}^G(m) < 0 \) for all \( m > 0 \). Then by the facts that \( \hat{u}(m) = \min[\hat{u}^C(m), \hat{u}^G(m)] \) and \( \hat{u}(0) = 0 \), it is optimal to choose a zero quota when contestants are strategic. The result that, when condition (24) is violated, a challenge equilibrium exists at \( m = m^*_M \) follows from the same argument as used in the proof of part (iii) of Theorem 1. \( \square \)

Proof of Result 1. First consider part (i). Because \( \mu_S > \mu_W \) and because, by hypothesis, \( \theta \geq \sigma \) in retention contests, condition (24) is satisfied. Part (i) then follows immediately from (a) Proposition 1, (b) the satisfaction of condition (24), and (c) the equivalence between designer objective given by (22) and designer objective in retention contests given by (25).

Next, consider part (ii). Because \( \tilde{S}_n \sim \text{Binomial}(n, \theta) \), where \( \tilde{S}_n \) denotes the number of

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\(^{29}\)If \( B(m^*_M; n, \theta) = 1 - \sigma \), then both \( m^*_M \) and \( m^*_M - 1 \), where \( m^*_M \) is given by (23), maximize \( \hat{u}^C \). Since, in the case in which two adjacent quotas are both optimal, the optimal quota is defined as the larger of the two, the optimal quota is \( m^*_M \).
strong workers, by the law of large numbers,
\[ \frac{S_n}{n} \xrightarrow{a.s.} \theta \quad \text{as } n \to \infty. \] (A-37)

Thus, asymptotically, absent risk-taking, the optimal fraction of retained workers equals \( \theta \), i.e.,
\[ \lim_{n \to \infty} m_M^*(n)/n = \theta. \]

Let \( \gamma = m/n \) be the retention rate. We can rewrite \( p_C^o \), using the first expression in the last line of (6), as follows:
\[ p_C^o = \frac{\mathbb{E}[\max[\gamma n - \tilde{S}_n, 0]]}{n(1 - \theta)} = \frac{1}{1 - \theta} \mathbb{E} \left[ \max \left[ \frac{\gamma - \tilde{S}_n}{n}, 0 \right] \right]. \] (A-38)

Equations (A-37) and (A-38) imply that
\[ p_C^o \to \max \left[ \frac{\gamma - \theta, 0}{1 - \theta} \right] \quad \text{as } n \to \infty. \] (A-39)

We can rewrite \( p_G^o \) using equation (10) as
\[ p_G^o = \gamma \left( \frac{\mu_W}{\theta \mu_S + (1 - \theta) \mu_W} \right). \] (A-40)

Equations (A-39) and (A-40) imply that
\[ \text{as } n \to \infty, \quad p_C^o > (=)(<) p_G^o \quad \text{if } \gamma > (=)(<) \theta + (1 - \theta)(\mu_W/\mu_S). \] (A-41)

Thus, when \( \gamma = \theta \), \( p_C^o < p_G^o \) as \( n \to \infty \). This implies, by Lemma 1, that asymptotically, challenge equilibria exist when the retention rate equals \( \theta \). Note that, as has been shown, asymptotically, the optimal retention rate when workers are non-strategic equals \( \theta \). Thus, given that, in retention contests, \( \theta \geq \sigma \), implying the satisfaction of condition (24), Proposition 1 implies that, asymptotically, when workers are strategic, the optimal retention rate must equal the cutoff rate such that challenge equilibria exist when the retention rate is below this cutoff while concession equilibria exist when the retention rate is above this cutoff. Hence, by Lemma 1, asymptotically, when workers are strategic, the optimal retention rate equals the one such that, at this rate, \( p_C^o = p_G^o \). Thus, by (A-41), asymptotically, the optimal retention rate equals \( \theta + (1 - \theta)(\mu_W/\mu_S) \) when workers are strategic. Part (ii) thus follows.

\[ \square \]

\textit{Proof of Proposition 2.} In a symmetric equilibrium, at the effort stage, each type-\( t \) contestant, \( t = S, W \), chooses the same capacity \( \mu_t \). Note that, in any symmetric equilibrium, it must be that \( \mu_t > 0, t = S, W \). This is because, if one type chose zero capacity, given that, with a positive probability, all contestants are of this type, with a positive probability, all contestants would choose zero capacity and have zero performance. In this case, any contestant of this type would be strictly better off choosing \( \epsilon > 0 \) capacity and a performance level equal to \( \epsilon \). The cost of choosing \( \epsilon > 0 \) capacity can be made arbitrarily small by shrinking \( \epsilon \) to zero while, for all positive \( \epsilon \), no matter how small, having \( \epsilon > 0 \) performance would generate a gain that is bounded below by a strictly positive number.

Let \( F_t(\cdot; \mu_W, \mu_S) \) be a type-\( t \) contestant’s performance distribution, \( t = S, W \), and \( P(\cdot; \mu_W, \mu_S) \) be the probability-of-winning function in a symmetric equilibrium of the subgame starting from
the risk-taking stage, when, at the effort stage, all type-τ contestants choose μr > 0. Let ν > 0 be the reward from being selected. Normalize the reward from being deselected to 0.

As shown by Figure 2 and the argument around it, P is concave.30 Because taking no risk is a best reply to a concave P, by choosing capacity μ, a contestant’s expected reward is given by νP(μ; μW, μS). In a symmetric equilibrium, it must be that a type-τ ∈ {S,W} contestant’s expected payoff, νP(μ; μW, μS) − (μa/a), is maximized at μ = μr. Because P is concave while the cost functions are strictly convex and because both P and the cost functions are continuous, for each t = S, W, μr is a best reply to Pt(·; μW, μS) for a type-τ contestant if and only if μr satisfies the following first-order condition:

\[ νP'(μr; μW, μS) = \frac{αμr^{α-1}}{a_r}. \]  

(A-42)

Note that, in any symmetric equilibrium, it must be that μS > μW. This is because, if, to the contrary, μS ≤ μW, the fact that aW < aS would imply that αμSα−1/aW > αμWα−1/aS. Thus, by (A-42), it would have to be that P(μW; μW, μS) > P(μS; μW, μS), which, given concavity of P, could only happen if μW < μS, contradicting the hypothesis that μS ≤ μW.

By Lemma 1, conditioned on that strong contestants choose μS and weak contestants choose μW in the effort stage, where μS > μW, in the subgame starting from the risk-taking stage, either concession or challenge equilibria exist. The next two results will be useful for our proof.

**Result A-2.** Define pO C and pO G as in (6) and (10), respectively. Define pS C as

\[ pS_C := \frac{1}{θ} \left( \frac{m}{n} - (1 - θ)pO_C \right). \]

(A-43)

Then pO C > (=)(<) pO G if and only if μS/μW > (=)(<) pS C / pO C.

**Proof.** Note that

\[ pO_C - pO_G = pO_C - \frac{m}{n} \left( \frac{μW}{θμS + (1 - θ)μW} \right) = pO_C \left( \frac{θpS_C + (1 - θ)pO_C}{θμS + (1 - θ)μW} \right) = \frac{θμW pO_C}{θμS + (1 - θ)μW} \left( \frac{μS}{μW} - \frac{pS_C}{pO_C} \right), \]

(A-44)

where the first equality follows from the definition of pO G given in (10), the second from the fact that pS C given in (A-43) satisfies that

\[ θpS_C + (1 - θ)pO_C = \frac{m}{n}, \]

(A-45)

and the last from simplifying the expression. By (A-44), the sign of pO C − pO G equals the sign of (μS/μW) − (pS C / pO C). The result thus follows.

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30Concavity of P holds even if μS ≤ μW, because if μS < μW, we can simply treat the high-ability type as the weak type and the low-ability type as the strong type at the risk-taking stage and all the arguments used in the proof of Lemma 1 apply. If μS = μW, one can treat each contestant as of the same type at the risk-taking stage by treating either θ = 0 or θ = 1. The proof of Lemma 1 does not rely on the value of θ. Thus, concavity of P still holds when μS = μW.
Result A-2, combined with Lemma 1, implies that in the subgame starting from the risk-taking stage, concession (challenge) equilibria exist if and only if $\mu_s/\mu_w \geq (\leq) p_s^C/p_o^C$. The next result allows us to compute $P'$, which will enable us to further apply equation (A-42).

**Result A-3.** Suppose that, in the effort stage, each type-$t \in \{S, W\}$ contestant chooses capacity $\mu_t$ and $\mu_S > \mu_W > 0$. Then the probability-of-winning function, $P(t; \mu_W, \mu_S)$, in any symmetric equilibrium of the subgame starting from the risk-taking stage satisfies the following conditions:

i. if $p_o^C < p_o^G$ (or equivalently, $\mu_S/\mu_W < p_s^C/p_o^C$), where $p_o^C$, $p_o^G$, and $p_s^C$ are given by (6), (10), and (A-43) respectively, then the subgame has challenge equilibria. In all of these challenge equilibria,

$$P(x; \mu_W, \mu_S) = \min \left[ \frac{m}{n(\theta \mu_S + (1-\theta)\mu_W)} x, 1 \right].$$  \hspace{1cm} (A-46)

ii. If $p_o^C \geq p_o^G$ (or equivalently, $\mu_S/\mu_W \geq p_s^C/p_o^C$), then the subgame has a unique concession equilibrium. In this concession equilibrium,

$$P(x; \mu_W, \mu_S) = \begin{cases} \beta_W x & x \in [0, \tilde{x}] \\ \alpha_S + \beta_S x & x \in [\tilde{x}, \hat{x}], \\ 1 & x \geq \hat{x} \end{cases}$$ \hspace{1cm} (A-47)

where $\max \text{Supp}_W = \min \text{Supp}_S = \tilde{x}$, and $\beta_W$, $\tilde{x}$, $\alpha_S$, $\beta_S$, and $\hat{x}$ are determined by contest parameters as follows:

$$\beta_W = p_o^C/\mu_W$$  \hspace{1cm} (A-48)

$$\tilde{x} = \bar{p}\mu_W/p_o^C$$ \hspace{1cm} (A-49)

$$\alpha_S = \frac{\bar{p}(\mu_S - (p_s^C \mu_W/p_o^C))}{\mu_S - (\bar{p}\mu_W/p_o^C)}$$ \hspace{1cm} (A-50)

$$\beta_S = \frac{p_s^C - \bar{p}}{\mu_S - (\bar{p}\mu_W/p_o^C)}$$ \hspace{1cm} (A-51)

$$\hat{x} = \frac{(1-\bar{p})\mu_S - (1-p_s^C)(\bar{p}\mu_W/p_o^C)}{p_o^C - \bar{p}}$$ \hspace{1cm} (A-52)

with $p_o^C$, $p_s^C$, and $\bar{p}$ given, respectively, by (6), (A-43), and

$$\bar{p} = \sum_{i=n-m}^{n-1} \binom{n-1}{i} (1-\theta)^i \theta^{n-1-i}.$$ \hspace{1cm} (A-53)

**Proof.** (i): The existence of challenge and the non-existence of concession equilibria when $p_o^C < p_o^G$ follow from Lemma 1. By Result A-2, $p_o^C < p_o^G$ is equivalent to $\mu_S/\mu_W < p_s^C/p_o^C$. As shown by Figure 2 and the argument around it, in challenge equilibria, there exists $\beta > 0$ such that $P(x; \mu_W, \mu_S) = \min[\beta x, 1]$. Given concavity of $P$, choosing a deterministic performance level equal to $\mu_W$ is a best response for a weak contestant in the risk-taking stage. Thus, given
that in challenge equilibria, a weak contestant’s probability of winning equals \( p^G_G \), where \( p^G_G \) is given by (10), it must be that \( P(\mu_W; \mu_W, \mu_S) = p^G_G \). Hence, \( \min[\beta \mu_W, 1] = p^G_G \), implying that
\[
\beta = \frac{m}{n(\theta \mu_S + (1 - \theta)\mu_W)}.
\]
Part (i) thus follows.

(ii): The existence of concession and the non-existence of challenge equilibria when \( p^C_C \geq p^G_G \) follow from Lemma 1. By Result A-2, \( p^C_C \geq p^G_G \) is equivalent to \( \mu_S / \mu_W \geq p^C_s / p^C_o \). As shown by Figure 2 and the argument around it, in concession equilibria, \( P \) must have the form given by (A-47), with \( \text{max Supp}_W = \text{min Supp}_S \).

Now we show that the five constants, \( \beta_W, \bar{x}, \alpha_S, \beta_S \), and \( \hat{x} \), must satisfy equations (A-48)–(A-52) in a concession equilibrium. First, continuity of \( P \), combined with (A-47), implies that
\[
\beta_W \bar{x} = \alpha_S + \beta_S \bar{x} \quad (A-54)
\]
\[
\alpha_S + \beta_S \bar{x} = 1. \quad (A-55)
\]
Next, concavity of \( P \) implies that, for any type \( t \in \{S, W\} \) contestant, choosing a deterministic performance level equal to \( \mu_t \) is a best response in the risk-taking stage. Note that \( p^C_C \), given in (6), denotes a weak contestant’s probability of winning in a concession equilibrium, and a strong contestant’s probability of winning has a relationship with a weak contestant’s probability of winning through equation (2). Thus, \( p^C_s \), given by (A-43), denotes a strong contestant’s probability of winning in a concession equilibrium. Thus, in a concession equilibrium, \( P(\mu_W; \mu_W, \mu_S) = p^C_C \) and \( P(\mu_S; \mu_W, \mu_S) = p^C_s \), which implies, given equation (A-47) and the fact that \( \text{max Supp}_W = \text{min Supp}_S = \bar{x} \) in a concession equilibrium, that
\[
\beta_W \mu_W = p^C_C \quad (A-56)
\]
\[
\alpha_S + \beta_S \mu_S = p^C_s. \quad (A-57)
\]
Finally, because, in a concession equilibrium, \( \text{max Supp}_W = \text{min Supp}_S = \bar{x} \), and because no contestant places point mass, for a given contestant, if her performance equals \( \bar{x} \), she will outperform all weak rivals but be outperformed by all strong rivals. Given that each rival is strong with probability \( \theta \) and rival types are independent, the given contestant’s probability of winning by having performance equal to \( \bar{x} \) in a concession equilibrium must equal \( \bar{p} \) given by (A-53). Thus, in a concession equilibrium, it must be that \( P(\bar{x}; \mu_W, \mu_S) = \bar{p} \), which, by (A-47), implies that
\[
\beta_W \bar{x} = \bar{p}. \quad (A-58)
\]
Equations (A-54)–(A-58) imply equations (A-48)–(A-52), and the result follows.

By Result A-3 and by the fact that \( p^C_s / p^C_o \) does not depend on \( \mu_S \) or \( \mu_W \), the two endogenous variables in the capacity acquisition model, to prove the proposition, it suffices to show that
1. when \( (a_S/a_W)^{1/(\alpha - 1)} < p^C_s / p^C_o \), there exist challenge equilibria but no concession equilibria.
In these challenge equilibria, in the effort stage, the capacity levels chosen by the two types satisfy that \( \mu_S / \mu_W = (a_S/a_W)^{1/(\alpha - 1)} \).
(2) When \((a_s/a_w)^{1/(a-1)} \geq p_s^C/p_o^C\), there exists a concession equilibrium but no challenge equilibrium. In this concession equilibrium, in the effort stage, the capacity levels chosen by the two types satisfy that \(\mu_S/\mu_W \geq (a_s/a_w)^{1/(a-1)}\).

In what follows, we prove the proposition by establishing (1) and (2). Note that challenge equilibria exist if and only if (i) the choice of \(\mu_S\) and \(\mu_W\) satisfies equation (A-42), where \(P\) is given by equation (A-46), and (ii) \(\mu_S/\mu_W < p_s^C/p_o^C\). In fact, the satisfaction of these conditions is equivalent to the satisfaction of the condition that \((a_s/a_w)^{1/(a-1)} < p_s^C/p_o^C\). To see this, note that, when \(P\) is given by (A-46), equation (A-42) implies that

\[
\frac{m v}{n (\theta \mu_s + (1 - \theta) \mu_w)} = \frac{\alpha \mu_t^{\alpha-1}}{a_t}, \quad t = S, W, \quad (A-59)
\]

which further implies that

\[
\mu_W = \left[ \frac{m v a_w}{n \alpha \left( \theta \left( \frac{a_s}{a_w} \right)^{\frac{1}{\alpha-1}} + 1 - \theta \right)} \right]^{\frac{1}{\alpha}}, \quad \mu_S = \left[ \frac{m v a_s \left( \frac{a_s}{a_w} \right)^{\frac{1}{\alpha-1}}}{n \alpha \left( \theta \left( \frac{a_s}{a_w} \right)^{\frac{1}{\alpha-1}} + 1 - \theta \right)} \right]^{\frac{1}{\alpha}}, \quad (A-60)
\]

and

\[
\frac{\mu_S}{\mu_W} = \left( \frac{a_s}{a_w} \right)^{\frac{1}{\alpha-1}}. \quad (A-61)
\]

By (A-61), \(\mu_S/\mu_W < p_s^C/p_o^C\) if and only if \((a_s/a_w)^{1/(a-1)} < p_s^C/p_o^C\). Thus, challenge equilibria exist if and only \((a_s/a_w)^{1/(a-1)} < p_s^C/p_o^C\), with the endogenous capacity levels and the endogenous capacity ratio given by (A-60) and (A-61) respectively.

Next, consider concession equilibria. A concession equilibrium exists if and only if (i) the choice of \(\mu_S\) and \(\mu_W\) satisfies equation (A-42), where \(P\) is given by equation (A-47), and (ii) \(\mu_S/\mu_W \geq p_s^C/p_o^C\). Note that, when \(P\) is given by (A-47), equations (A-42), (A-48), and (A-51) imply that

\[
\frac{\alpha \mu_W^{\alpha-1}}{a_W} = \frac{p_s^C}{\mu_W} \quad (A-62)
\]

\[
\frac{\alpha \mu_S^{\alpha-1}}{a_s} = \frac{p_s^C - \bar{p}}{\mu_S - (\bar{p} \mu_W/p_o^C)}. \quad (A-63)
\]

In what follows, we show that, when \(\mu_W\) and \(\mu_S\) satisfy equations (A-62) and (A-63), \(\mu_S/\mu_W \geq p_s^C/p_o^C\) if and only if \((a_s/a_w)^{1/(a-1)} \geq p_s^C/p_o^C\).

Divide equation (A-63) by equation (A-62) on both sides and rearrange the result. This yields

\[
\left( \frac{\mu_S}{\mu_W} \right)^{\alpha-1} - \frac{a_s}{a_W} \left( \frac{p_s^C - \bar{p}}{\frac{\mu_s}{\mu_W} p_o^C - \bar{p}} \right) = 0. \quad (A-64)
\]

Let \(r = \mu_S/\mu_W\) and \(z = a_s/a_W\). Treat the left-hand side of (A-64) as a function of \(r\) and \(z\) and denote it by \(\mathcal{X}\), i.e.,

\[
\mathcal{X}(r, z) := r^{\alpha-1} - z \left( \frac{p_s^C - \bar{p}}{r p_o^C - \bar{p}} \right). \quad (A-65)
\]
By (A-65),
\[
\mathcal{K} \left( r = \frac{p^C}{p^C_o}, z = (\frac{p^C}{p^C_o})^{-1} \right) = 0. \tag{A-66}
\]

Note that \( p^C_s \), given by (A-43), is a strong contestant’s probability of winning in a concession equilibrium, i.e., \( p^C_s \) represents a contestant’s probability of winning if the contestant always beats weak rivals and shares the same winning probability with strong rivals. Also note that \( \tilde{p} \), given by (A-53), is a contestant’s probability of winning if the contestant always beats weak rivals but is always beaten by strong rivals. Thus, it is clear that
\[
p^C_s > \tilde{p}. \tag{A-67}
\]

Equation (A-65) thus implies that
\[
\text{for any fixed } z \geq 0, \mathcal{K} \text{ is increasing in } r \text{ for } r \geq \frac{p^C_s}{p^C_o}, \tag{A-68}
\]

&
\[
\text{for any fixed } r \geq \frac{p^C_s}{p^C_o}, \mathcal{K} \text{ is decreasing in } z \text{ for } z \geq 0. \tag{A-69}
\]

Hence,
\[
\forall r \geq \frac{p^C_s}{p^C_o} \& 0 \leq z < (\frac{p^C_s}{p^C_o})^{-1}, \mathcal{K} (r, z) \geq \mathcal{K} (\frac{p^C_s}{p^C_o}, z) \]
\[
> \mathcal{K} \left( \frac{p^C_s}{p^C_o}, (\frac{p^C_s}{p^C_o})^{-1} \right) = 0, \tag{A-70}
\]

where the first inequality follows from (A-68), the second inequality from (A-69), and the last equality from (A-66). By definition, \( r = \mu_S/\mu_W \) and \( z = a_S/a_W \). Thus, equation (A-70) implies that, when \( a_S/a_W < (p^C_s/p^C_o)^{-1} \) (i.e., when \( (a_S/a_W)^{1/(\alpha-1)} < (p^C_s/p^C_o) \)), there exists no capacity ratio, \( \mu_S/\mu_W \), that both solves equation (A-64) and satisfies that \( \mu_S/\mu_W \geq p^C_s/p^C_o \). Because the existence of a concession equilibrium requires the existence of \( \mu_S/\mu_W \geq p^C_s/p^C_o \) that solves equation (A-64), when \( (a_S/a_W)^{1/(\alpha-1)} < (p^C_s/p^C_o) \), no concession equilibrium exists.

Equations (A-66) and (A-69) imply that
\[
\text{for any fixed } z \geq (\frac{p^C_s}{p^C_o})^{-1}, \mathcal{K} (\frac{p^C_s}{p^C_o}, z) \leq \mathcal{K} (\frac{p^C_s}{p^C_o}, (\frac{p^C_s}{p^C_o})^{-1}) = 0. \tag{A-71}
\]

Equations (A-65) and (A-67) imply that
\[
\text{for any fixed } z > 0, \mathcal{K} (r, z) \to \infty \text{ as } r \to \infty. \tag{A-72}
\]

Hence, by equations (A-68), (A-71), and (A-72), for any fixed \( z \geq (\frac{p^C_s}{p^C_o})^{-1} \), there exists a unique \( r^o \geq \frac{p^C_s}{p^C_o} \) such that \( \mathcal{K} (r = r^o, z) = 0. \) Thus, by the definitions of \( r, z, \) and \( \mathcal{K} \), when \( a_S/a_W \geq (p^C_s/p^C_o)^{-1} \) (i.e., when \( (a_S/a_W)^{1/(\alpha-1)} \geq (p^C_s/p^C_o) \)), there exists a unique capacity ratio, \( \mu_S/\mu_W \), that both solves equation (A-64) and satisfies that \( \mu_S/\mu_W \geq p^C_s/p^C_o \). Because a concession equilibrium exists if there exists a pair of \( \mu_W \) and \( \mu_s \) that solves equations (A-62) and (A-64) and satisfies that \( \mu_S/\mu_W \geq (p^C_s/p^C_o) \), and because there clearly exists a unique \( \mu_W \) that solves equation (A-62), when \( (a_S/a_W)^{1/(\alpha-1)} \geq (p^C_s/p^C_o) \), a concession equilibrium exists.

Therefore, a concession equilibrium exists if and only if \( (a_S/a_W)^{1/(\alpha-1)} \geq (p^C_s/p^C_o) \).

**Proof of Result 2.** Equation (A-60) shows each type of contestant’s effort choice in a challenge equilibrium. In equation (A-60), \( \nu \) denotes the reward to a selected contestant. Total rewards
equal \(mv\), which is fixed if the reward budget is fixed. Inspection of equation (A-60) reveals that (a) fixing \(mv\) and \(n\) fixes each type’s effort choice in a challenge equilibrium and (b) fixing \(mv\) while increasing \(n\) reduces each type’s effort choice in a challenge equilibrium. Part (i) follows from (a) and part (ii) from (b).

Proof of Proposition 3. Let \(\bar{x}\) be the exogenous upper bound on performance. Suppose that \(\bar{x} \geq \mu_S\). Then any symmetric equilibrium has the following properties:

**No point mass on zero performance** This follows from the same contradiction argument used in the proof of Lemma 1 for showing no point mass on zero performance when there is no exogenous upper bound on performance.

**No point mass on any performance level on \((0, \bar{x})\)** This follows from applying the contradiction argument used in the proof of Lemma 1 for showing no point mass on any strictly positive performance in the case of no exogenous upper bound on performance to any performance level on \((0, \bar{x})\).

**\(P\) is continuous over \([0, \bar{x})\).** This follows from the fact that, as established above, no contestant places point mass on any \(x < \bar{x}\).

Thus, only \(\bar{x}\) can possibly be a discontinuity point for \(P\). Because contestants are not allowed to place weight on performance above \(\bar{x}\) and because \(P\) is continuous over \([0, \bar{x})\), the set of optimal fair gambles is closed. Thus, the support of a contestant’s equilibrium performance distribution is contained in the set of optimal fair gambles for the contestant. This fact, combined with the argument used around Figure 1 in the main text, implies that Result A-1, which holds in the case of no exogenous upper bound on performance, also holds when there is an exogenous upper bound, \(\bar{x} \geq \mu_S\), on performance.

Result A-4. Result A-1 still holds when there is an exogenous upper bound, \(\bar{x} \geq \mu_S\), on performance.

**The lower bound of the support of \(P\) is zero.** By equation (15), which holds regardless of whether performance is exogenously bounded or not, there exists at least one type, say \(t\), such that \(\min \text{Supp}_t = \min \text{Supp} P\). Because, as established above, no contestant places point mass on any \(x < \bar{x}\), and because \(\mu_W < \mu_S \leq \bar{x}\), weak contestants place no point mass on \(\mu_W\). Thus, given that a weak contestant’s expected performance equals \(\mu_W\), it must be that \(\min \text{Supp}_W < \mu_W\). This implies, by equation (15), that \(\min \text{Supp} P < \mu_W\). Hence, given that \(\min \text{Supp}_t = \min \text{Supp} P\) and \(\mu_W < \mu_S\), it must be that \(\min \text{Supp}_t < \mu_t\), which implies, given that the mean of \(F_t\) equals \(\mu_t\), the existence of \(x^o > \mu_t\) such that \(x^o \in \text{Supp}_t\). The result that \(\min \text{Supp} P = 0\) then follows from the same contradiction argument used in the proof of Lemma 1 for showing zero as the lower bound of the support of \(P\) in the case of no exogenous upper bound on performance.

**\(\text{Supp} P\setminus \{\bar{x}\}\) is a connected set.** Otherwise, there would exist \(0 \leq x' < x'' < \bar{x}\) such that \([x', x'']\cap \text{Supp} P = \{x', x''\}\). In this case, by equation (15), there would exist at least one type, say \(t\), such that \((x', x'') \cap \text{Supp}_t = \{x''\}\). Because, by hypothesis, \(x'' < \bar{x}\), and because, as has been
established, no contestant places point mass on any \( x \in [0, \bar{x}) \), \( (x', x'') \cap \text{Supp}_t = \{x''\} \) would imply the existence of \( \varepsilon > 0 \) such that \([x'', x' + \varepsilon] \in \text{Supp}_t\). The connectedness of \( \text{Supp} P \setminus \{\bar{x}\} \) then follows from a similar contradiction argument used in the proof of Lemma 1 for showing the connectedness of \( \text{Supp} P \) in the case of no exogenous upper bound on performance.

These properties of \( P \), combined with Result A-4, allow us to show the following result:

**Result A-5.** In any challenge equilibrium, all pairs \((x, P(x))\) such that \( x \in \text{Supp} P \) must be collinear.

**Proof.** First, note that, in any challenge equilibrium, it must be that \( \text{Supp}_w \cap \text{Supp}_s \neq \emptyset \). This is because, by equation (15) and the facts that \( \text{Supp}_w \) and \( \text{Supp}_s \) are closed, if \( \text{Supp}_w \cap \text{Supp}_s = \emptyset \), it had to be that one type places all weight on \( \bar{x} \) and the other type places all weight over some interval lying strictly below \( \bar{x} \). This would imply that the former type always bests the latter type, contradicting the fact that, in a challenge equilibrium, each type has a positive probability of besting the other type.

Next, given that \( \text{Supp}_w \cap \text{Supp}_s \neq \emptyset \), there are only two cases to consider: (1) the case in which \( \text{Supp}_w \cap \text{Supp}_s \) contains more than one point and (2) the case in which \( \text{Supp}_w \cap \text{Supp}_s \) consists of a single point. We first consider case (1). Suppose that, in a challenge equilibrium, there exist two distinct points \( x' \) and \( x'' \) such that \( x', x'' \in \text{Supp}_w \cap \text{Supp}_s \). Then by Result A-4, all pairs \((x, P(x))\) such that \( x \in \text{Supp}_t \), \( t = s, w \), are collinear with \((x', P(x'))\) and \((x'', P(x''))\). In this case, by equation (15), all pairs \((x, P(x))\) such that \( x \in \text{Supp} P \) must be collinear.

Now consider case (2). Suppose that, in a challenge equilibrium, \( \text{Supp}_w \cap \text{Supp}_s = x^o \). In case (2), there are two subcases to consider: (a) \( x^o = \bar{x} \), and (b) \( x^o < \bar{x} \). First consider the subcase in which \( x^o = \bar{x} \) (i.e., \( \text{Supp}_w \cap \text{Supp}_s = \bar{x} \)). Note that, by equation (15) and the connectedness of \( \text{Supp} P \setminus \{\bar{x}\}, \text{Supp}_w \cup \text{Supp}_s \setminus \{\bar{x}\} \) is connected. Because, by hypothesis, \( \text{Supp}_w \cap \text{Supp}_s = \bar{x} \), it must be that \( \text{Supp}_w \cap \text{Supp}_s \setminus \{\bar{x}\} = \emptyset \). Thus, given that \( \text{Supp}_w \cup \text{Supp}_s \setminus \{\bar{x}\} \) is connected and both \( \text{Supp}_w \) and \( \text{Supp}_s \) are closed, it must be that either \( \text{Supp}_w \setminus \{\bar{x}\} = \emptyset \) or \( \text{Supp}_s \setminus \{\bar{x}\} = \emptyset \). Because \( \text{Supp}_w \setminus \{\bar{x}\} = \emptyset \) would violate the weak type’s capacity constraint, it must be that \( \text{Supp}_s \setminus \{\bar{x}\} = \emptyset \) (clearly, this subcase can happen only when \( \bar{x} = \mu_S \)).

Given \( \text{Supp}_s = \bar{x} \), for this equilibrium to be a challenge equilibrium, it must be that \( \bar{x} \in \text{Supp}_w \), implying that \( \text{Supp}_s \subset \text{Supp}_w \). Thus, by equation (15), \( \text{Supp} P = \text{Supp}_w \). Hence, given that, by Result A-4, all pairs \((x, P(x))\) such that \( x \in \text{Supp}_w \) are collinear, all pairs \((x, P(x))\) such that \( x \in \text{Supp} P \) must be collinear.

Next, consider the subcase in which \( x^o < \bar{x} \). In this subcase, it must be that at least one type places point mass on \( \bar{x} \). This is because, otherwise, \( \text{Supp} P \) would have to be connected. This would imply, given \( \text{Supp}_w \cap \text{Supp}_s = x^o < \bar{x} \) and the fact that no contestant places point mass on any \( x < \bar{x} \), that the equilibrium under consideration is not a challenge equilibrium but a concession equilibrium where \( \max \text{Supp}_w = \min \text{Supp}_s \), a contradiction.

Thus, at least one type, say \( t \), places point mass on \( \bar{x} \). Then the hypothesis that \( \text{Supp}_w \cap \text{Supp}_s = x^o < \bar{x} \) implies that the other type, denoted by \(-t\), must not place point mass on \( \bar{x} \). Then, given
the facts that (a) \( \text{Supp} P \setminus \{\tilde{x}\} \) is connected with \( \min \text{Supp} P = 0 \), (b) no contestant places point mass on any \( x < \tilde{x} \), and (c) in a challenge equilibrium, each type has a positive probability of besting the other type, there must exist \( x' \in (x^o, \tilde{x}) \) such that

\[
\text{Supp}_t = [0, x^o] \cup \{\tilde{x}\} \quad \& \quad \text{Supp}_o = [x^o, x'].
\]  
(A-73)

Then by Result A-4,

all pairs \( (x, P(x)) \) such that \( x \in [0, x^o] \cup \{\tilde{x}\} \) are collinear  
(A-74)

& all pairs \( (x, P(x)) \) such that \( x \in [x^o, x'] \) are collinear.  
(A-75)

By (A-74) and (A-75), over \( [0, x'] \), \( P \) is piecewise linear, with \( x^o \) being the only possible kink. Note that the slope of \( P \) over \( (0, x^o) \) must be no less than the slope of \( P \) over \( (x^o, x') \). This is because, otherwise, \( P \) would be convex and nonlinear over \( [0, x'] \), in which case, by Jensen’s inequality, type \( -t \) would be strictly better off deviating to a fair gamble between 0 and \( x' \), a contradiction. Note also that the slope of \( P \) over \( (0, x^o) \) must be no greater than the slope of \( P \) over \( (x^o, x') \). This is because, by (A-74), the slope of \( P \) over \( (0, x^o) \) equals the slope of the line connecting the two points, \( (x^o, P(x^o)) \) and \( (\tilde{x}, P(\tilde{x})) \). Thus, if this slope were strictly greater than the slope of \( P \) over \( (x^o, \tilde{x}) \), type \( -t \) would be strictly better off deviating to a fair gamble between \( x^o \) and \( \tilde{x} \), a contradiction. Therefore, the slope of \( P \) over \( (0, x^o) \) has to equal the slope of \( P \) over \( (x^o, x') \). This implies, by equations (15), (A-73), (A-74), and (A-75), that all pairs \( (x, P(x)) \) such that \( x \in \text{Supp} P \) are collinear.

The above analysis exhausts all possible configurations of challenge equilibria and the result thus follows.  
\[ \square \]

Result A-5 and the fact that \( \min \text{Supp} P = 0 \) imply that, in any challenge equilibrium (if exists), it is a best response for a weak contestant to play a strategy that gives her a strong type’s performance distribution with probability \( \mu_w / \mu_s \) and zero performance with the complementary probability. Because no contestant places point mass on 0, we have \( P(0) = 0 \). Thus, in a challenge equilibrium, \( p_w = (\mu_w / \mu_s) p_s \). Then, by equation (2), in a challenge equilibrium, a weak contestant’s probability of winning equals \( p_o^G \) given by (10). Note that a weak contestant’s probability of winning in a concession equilibrium still equals \( p_o^C \) given by (6). Because, as shown in Section 2 by an argument that does not depend on whether there exists an upper bound, \( \tilde{x} \geq \mu_s \), on performance, \( p_o^G \) and \( p_o^C \) are two lower bounds of \( p_w \). Thus, given that every symmetric equilibrium is either a concession or a challenge equilibrium, if \( p_o^C \geq (\text{<}) p_o^G \), only concession (challenge) equilibria exist. The proposition thus follows.  
\[ \square \]

**Proof of Result 3.** Note that, in challenge equilibria, the probability that a selected contestant is weak, denoted by \( P[W|\text{selected}] \), is given by

\[
P[W|\text{selected}] = \frac{(1 - \theta) p_w}{\theta p_s + (1 - \theta) p_w} = \frac{n(1 - \theta) p_o^G}{m} = \frac{(1 - \theta) \mu_w}{\theta \mu_s + (1 - \theta) \mu_w},
\]  
(A-76)

where the first equality follows from Bayes rule, the second from equation (2) and the fact that, in challenge equilibria, \( p_w = p_o^G \), and the last from equation (10). By equation (A-76), the
$n$-contestant/$m$-winner contest and the $n$-contestant/$m'$-winner contest will produce the same $\mathbb{P}[W|\text{selected}]$ and, hence, the same $\mathbb{P}[S|\text{selected}] = 1 - \mathbb{P}[W|\text{selected}]$ as long as both contests have challenge equilibria. Thus, the expected number of weak selected contestants and the expected number of strong selected contestants produced by the “relaxed” selection policy will be the same as those produced by the original selection policy, implying the same designer welfare (22).
B. Construction of an equilibrium

Result B-1. If \( p_o^C \geq p_o^G \), where \( p_o^C \) and \( p_o^G \) are given in equations (6) and (10) respectively, there exists a unique concession equilibrium. In this equilibrium, the probability-of-winning function, \( P \), is given by (A-47), and \( F_W \) and \( F_S \) are given as follows: define \( \beta_w, \bar{x}, \alpha_S, \beta_S, \hat{x} \) by equations (A-48)–(A-52), respectively. Then define

\[
\phi(y) := \frac{1}{\beta_w} \sum_{i=n-m}^{n-1} \binom{n-1}{i} [(1-\theta)y]^i [1-(1-\theta)y]^{n-1-i}, \quad y \in [0, 1]
\]

\[
\zeta(y) := \frac{1}{\beta_S} \sum_{i=n-m}^{n-1} \binom{n-1}{i} [1-\theta + \theta y]^i [\theta(1-y)]^{n-1-i} - \alpha_S, \quad y \in [0, 1].
\]

\( \phi : [0, 1] \rightarrow [0, \bar{x}] \) and \( \zeta : [0, 1] \rightarrow [\bar{x}, \hat{x}] \) are both increasing, smooth, and continuous. Thus, their inverse functions, \( \phi^{-1} : [0, \bar{x}] \rightarrow [0, 1] \) and \( \zeta^{-1} : [\bar{x}, \hat{x}] \rightarrow [0, 1] \) exist. In the concession equilibrium, \( \text{Supp}_W = [0, \bar{x}] \) and \( \text{Supp}_F = [\bar{x}, \hat{x}] \), and over the corresponding support, \( F_W \) and \( F_S \) are given by

\[
F_W(x) = \phi^{-1}(x), \quad x \in [0, \bar{x}]; \quad F_S(x) = \zeta^{-1}(x), \quad x \in [\bar{x}, \hat{x}].
\]

**Proof.** Result A-3(ii) shows the unique form of the probability-of-winning function, \( P \), in a concession equilibrium. Note that, in a concession equilibrium, \( P \) must be weakly concave and increasing over its support. Thus, to construct a concession equilibrium when \( p_o^C \geq p_o^G \), we first show that, when \( p_o^C \geq p_o^G \), \( P \), given in Result A-3(ii), is weakly concave and increasing over its support. By Result A-3(ii), this is equivalent to showing that \( \beta_w \geq \beta_S > 0 \), where \( \beta_w \) and \( \beta_S \) are given by (A-48) and (A-51) respectively.

Note that, by Result A-2, when \( p_o^C \geq p_o^G \), \( \mu_S \geq p_S^C \mu_W / p_o^C \), implying that

\[
\mu_S - (\bar{p} \mu_W / p_o^C) \geq (\hat{p} - \bar{p}) \mu_W / p_o^C > 0,
\]

where the last inequality follows from (A-67). Equations (A-51), (A-67), and (B-4) imply that \( \beta_S > 0 \) and also that

\[
\beta_S = \frac{p_S^C - \bar{p}}{\mu_S - (\bar{p} \mu_W / p_o^C)} \leq \frac{p_S^C - \bar{p}}{(\hat{p} - \bar{p}) \mu_W / p_o^C} = \frac{p_o^C}{\mu_W} = \beta_w,
\]

where the last equality follows from (A-48). Thus, when \( p_o^C \geq p_o^G \), \( \beta_w \geq \beta_S > 0 \), implying that \( P \) given in Result A-3(ii) is weakly concave and increasing over its support.

Next, we show that \( F_W \) and \( F_S \), constructed in (B-3), jointly produce the form of \( P \) shown in Result A-3(ii) and are CDFs that satisfy their capacity constraints. Note that the performance distribution chosen by a contestant of unknown type is given by

\[
F(x) = \theta F_S(x) + (1 - \theta) F_W(x).
\]

In any symmetric equilibrium, no one places point mass. Thus, if a contestant has performance equal to \( x \), her probability of besting any given rival of unknown type equals \( F(x) \). To win a place, the contestant has to best at least \( (n-m) \) out of her \( (n-1) \) rivals, whose types are
unknown to her and whose performances are independent. Thus, in any symmetric equilibrium, a contestant’s probability-of-winning function, \( P \), has a relation to \( F \) given by

\[
P(x) = \sum_{i=n-m}^{n-1} \binom{n-1}{i} F(x)^i (1 - F(x))^{n-1-i}.
\]

By construction, \( \text{Supp}_W = [0, \bar{x}] \) and \( \text{Supp}_S = [\bar{x}, \bar{x}] \). Thus, by (B-5), \( F(x) = (1 - \theta) F_{W}(x) \) for \( x \in [0, \bar{x}] \), and \( F(x) = 1 - \theta + \theta F_{S}(x) \) for \( x \in [\bar{x}, \bar{x}] \). Hence, by equations (B-1), (B-2), and (B-6),

\[
P(x) = \begin{cases} 
\beta_W \phi \circ F_W(x) & x \in [0, \bar{x}] \\
\alpha_S + \beta_S \zeta \circ F_S(x) & x \in [\bar{x}, \bar{x}] \\
1 & x \geq \bar{x}
\end{cases}
\]

When \( F_W(x) = \phi^{-1}(x) \) for \( x \in [0, \bar{x}] \) and \( F_S(x) = \zeta^{-1}(x) \) for \( x \in [\bar{x}, \bar{x}] \), \( P \), given by (B-7), coincides with \( P \) given in Result A-3(ii). Thus, \( F_W \) and \( F_S \), constructed in (B-3), jointly produce the form of \( P \) shown in Result A-3(ii).

Now we verify that \( F_W \) and \( F_S \), constructed in (B-3), satisfy their capacity constraints. First, consider \( F_W \). Let \( \beta_W \) be the mean of \( F_W \) constructed in (B-3). Note that

\[
\hat{\mu}_W = \int_{0-}^{\bar{x}} x dF_W(x) = \int_{0-}^{1} F_W^{-1}(y) dy = \int_{0-}^{1} \phi(y) dy = \frac{1}{\beta_W} \int_{0-}^{1} \sum_{i=n-m}^{n-1} \binom{n-1}{i} [(1 - \theta)y]^i [1 - (1 - \theta)y]^{n-1-i} dy,
\]

where the third equality follows from the construction of \( F_W \) in (B-3) and the last from (B-1). Also note that

\[
\int_{0-}^{1} \sum_{i=n-m}^{n-1} \binom{n-1}{i} [(1 - \theta)y]^i [1 - (1 - \theta)y]^{n-1-i} dy = p_o^C,
\]

where \( p_o^C \) is given in (6). To see why (B-9) holds, note that the left-hand side of (B-9) can be interpreted as a weak contestant’s probability of winning if she concedes to all strong rivals and both she and her weak rivals play a uniform performance distribution on \([0, 1]\). In this hypothetical contest, given that the given weak contestant has no chance of besting strong rivals and shares the same probability of winning with weak rivals, the given contestant’s probability of winning must equal \( p_o^C \), a weak contestant’s probability of winning in a concession equilibrium.

Equations (B-8) and (B-9) imply that \( \hat{\mu}_W = p_o^C / \beta_W \). Thus, by (A-48), we must have \( \hat{\mu}_W = \mu_W \) and, hence, the construction of \( F_W \) satisfies \( W \)'s capacity constraint. Next, by a similar argument, we can also verify that \( F_S \), constructed in (B-3), satisfies \( S \)'s capacity constraint. We omit the detailed proof.

The above analysis verifies that, when \( p_o^C \geq p_o^C \), there exist \( F_W \) and \( F_S \), constructed in (B-3), that satisfy their capacity constraints and jointly produce an increasing and weakly concave \( P \) in the form of (A-47). The concavity of \( P \) ensures that the constructed strategies are best responses and thus sustain an equilibrium. \( \square \)
Result B-2. Suppose $p^C_o < p^G_o$, where $p^C_o$ and $p^G_o$ are given by (6) and (10) respectively. Then challenge equilibria exist. In all of these challenge equilibria, the probability-of-winning function, $P$, is given by (A-46). Moreover, there exist positive constants, $x^o$, $\rho_w$, and $\rho_s$, where $x^o < n(\theta \mu_s + (1 - \theta)\mu_w)/m$ and $\rho_w > \rho_s$, and distributions, $F^o_w$ and $F^o_s$, with $F^o_w$ supported by $[0, x^o]$ and $F^o_s$ supported by $[x^o, n(\theta \mu_s + (1 - \theta)\mu_w)/m]$, such that, in one of these challenge equilibria, each weak contestant plays $F^o_w$ with probability $\rho_w$ and plays $F^o_s$ with probability $1 - \rho_w$ and each strong contestant plays $F^o_w$ with probability $\rho_s$ and plays $F^o_s$ with probability $1 - \rho_s$ (the construction of such a challenge equilibrium is provided in the proof).

Proof. By Result A-3, in challenge equilibria, the probability-of-winning function, $P$, is the CDF of a uniform distribution given by equation (A-46). By Result A-2, $p^C_o < p^G_o$ is equivalent to $\mu_s/\mu_w < p^C_s/p^C_o$, where $p^C_s$ is given by (A-43). Note that neither $p^C_s$ nor $p^C_o$ depend on $\mu_w$ or $\mu_s$. To prove Result B-2, it suffices to construct a pair of CDFs, $F_s$ and $F_w$, that produce a uniform $P$ given by (A-46) and satisfy the specific characterization given in Result B-2 and their capacity constraints under the condition that $\mu_s/\mu_w < p^C_s/p^C_o$. Below we provide this construction.

Note that, when $\mu_s/\mu_w < p^C_s/p^C_o$, there always exists a unique pair of $\mu^o_i$ and $\mu^o_S$ such that

$$\mu^o_S > \mu_s > \mu_w > \mu^o_W > 0$$

(B-10)

$$\theta \mu^o_S + (1 - \theta) \mu^o_W = \theta \mu_s + (1 - \theta) \mu_w$$

(B-11)

$$\mu^o_S/\mu^o_W = p^C_s/p^C_o.$$  

(B-12)

Consider an auxiliary contest where weak contestants have capacity $\mu^o_w$ and strong contestants have capacity $\mu^o_S$. Because $\mu^o_w$ and $\mu^o_S$ satisfy equation (B-12), by Results A-2 and B-1, this auxiliary contest has a concession equilibrium and in this concession equilibrium, $P(x; \mu^o_w, \mu^o_S)$ is given by (A-47). Inspection of (A-46) and (A-47) reveals that, when $\mu^o_w$ and $\mu^o_S$ satisfy equation (B-12), we can equivalently express $P(x; \mu^o_w, \mu^o_S)$ using equation (A-46). Thus, by (B-5) and (B-6), if $F^o_w$ and $F^o_s$ represent, respectively, weak and strong contestants’ equilibrium performance distributions in this auxiliary contest (see Result B-1 for the construction of $F^o_w$ and $F^o_s$), $F^o_w$ and $F^o_s$ satisfy that

$$\sum_{i=n-m}^{n-1} \binom{n-1}{i} (\theta F^o_s(x) + (1 - \theta) F^o_w(x)) (1 - \theta F^o_s(x) - (1 - \theta) F^o_w(x))^{n-1-i}$$

$$= \min \left[ \frac{m}{n(\theta \mu^o_S + (1 - \theta) \mu^o_w)} x, 1 \right].$$

(B-13)

Now we verify that, when $\mu_s/\mu_w < p^C_s/p^C_o$, there exists a challenge equilibrium in which strong and weak contestants play as follows:

\[ W\text{-strategy} = \begin{cases} F^o_w & \text{w. p. } \rho_w \\ F^o_s & \text{w. p. } 1 - \rho_w \end{cases} \quad \text{S\text{-strategy} } = \begin{cases} F^o_w & \text{w. p. } \rho_s \\ F^o_s & \text{w. p. } 1 - \rho_s \end{cases}. \]

(B-14)
where

\[ \rho_w := \frac{\mu^o_S - \mu_W}{\mu^o_S - \mu^o_W} \quad \text{and} \quad \rho_S := \frac{\mu^o_S - \mu_S}{\mu^o_S - \mu^o_W}. \] (B-15)

First, note that, when \( \mu_S / \mu_W < p^c_S / p^C_o \), equation (B-10) holds, which, by (B-15), implies that

\[ 0 < \rho_S < \rho_w < 1. \] (B-16)

Next, note that \( F^o_W \) and \( F^o_S \) represent the equilibrium performance distributions played in the auxiliary contest, where weak contestants’ capacity equals \( \mu^o_w \) and strong contestants’ capacity equals \( \mu^o_S \). Thus, the mean of \( F^o_W \) equals \( \mu^o_w \) and the mean of \( F^o_S \) equals \( \mu^o_S \). Hence, by playing the strategies presented in (B-14), weak contestants’ mean performance equals \( \rho_w \mu^o_w + (1 - \rho_w) \mu^o_o \), which, given the definition of \( \rho_w \) in (B-15), equals \( \mu_w \), and strong contestants’ mean performance equals \( \rho_S \mu^o_w + (1 - \rho_S) \mu^o_o \), which, given the definition of \( \rho_S \) in (B-15), equals \( \mu_S \). Thus, the strategies presented in (B-14) satisfy the capacity constraints. By (B-16), these strategies give weak contestants a positive probability of besting strong contestants.

Finally, we show that the strategies presented in (B-14) jointly produce a uniform \( P \) given in (A-46). Note that, when the two types use the strategies presented in (B-14), the performance distribution played by a contestant of unknown type, \( F \), satisfies that

\[ F(x) = \theta F_S(x) + (1 - \theta) F_W(x) \]

\[ = (\theta \rho_S + (1 - \theta) \rho_w) F^o_W(x) + (1 - \theta \rho_S - (1 - \theta) \rho_w) F^o_S(x). \] (B-17)

Also note that

\[ \theta \rho_S + (1 - \theta) \rho_w = \theta \left( \frac{\mu^o_S - \mu_S}{\mu^o_S - \mu^o_W} \right) + (1 - \theta) \left( \frac{\mu^o_S - \mu_W}{\mu^o_S - \mu^o_W} \right) \]

\[ = \frac{\mu^o_S - (\theta \mu_S + (1 - \theta) \mu_W)}{\mu^o_S - \mu^o_W} = \frac{\mu^o_S - (\theta \mu^o_S + (1 - \theta) \mu^o_W)}{\mu^o_S - \mu^o_W} = 1 - \theta, \] (B-18)

where the first equality follows from (B-15) and the second last from (B-11). Equations (B-17) and (B-18) imply that

\[ F(x) = \theta F_S(x) + (1 - \theta) F_W(x) = \theta F^o_S(x) + (1 - \theta) F^o_W(x). \] (B-19)

Let \( P(\cdot; \mu_W, \mu_S) \) be the probability-of-winning function produced by \( F_W \) and \( F_S \). Equations (B-6), (B-13), and (B-19) imply that

\[ P(x; \mu_W, \mu_S) = \min \left[ \frac{m}{n (\theta \mu_S + (1 - \theta) \mu_W)} x, 1 \right]. \]

This further implies, given equation (B-11), that

\[ P(x; \mu_W, \mu_S) = \min \left[ \frac{m}{n (\theta \mu_S + (1 - \theta) \mu_W)} x, 1 \right]. \]

Thus, the strategies presented in (B-14) jointly produce a uniform \( P \) given in (A-46). The result thus follows. \( \square \)
C. Pool expansion by including less promising candidates

In Section 5, we studied the effect of risk-taking on the optimal size of the contestant pool. We showed that, with the selection quota fixed, if the contestant pool is sufficiently large, adding more contestants who are as likely to exhibit ability as the contestants in the original pool does not affect the expected number of strong selected contestants. Thus, a meritocratic contest designer has no incentive to expand the contestant pool if the pool is already sufficiently large. The next result shows that, in fact, expanding a large pool makes selection less meritocratic if the external candidates are less likely to exhibit ability than the contestants in the original pool.

Result C-1. Suppose that the designer can only expand the contestant pool by including external candidates whose prior quality (measured by $\theta$) is lower than the internal candidates’. If the contest with only the internal candidates has challenge equilibria, pool expansion strictly reduces designer welfare (19) in any symmetric equilibrium (“symmetric” in the sense that each type-$t \in \{S, W\}$ internal candidate plays the same strategy and each type-$t \in \{S, W\}$ external candidate plays the same strategy).

Result C-1 implies that, even without any direct cost of pool expansion, as long as the external candidates are ex-ante less promising than internal ones, a meritocratic designer strictly prefers “limiting the field” only to internal candidates if the internal competition already supports challenge equilibria. Result C-1 might shed some light on why many real-world selection contests limit participation by requiring, sometimes in a de facto way, candidates to have certain qualifications to be eligible for contest participation.

Proof of Result C-1. Let $\theta$ ($\theta'$) be the probability of being strong for each internal (external) candidate, where $\theta > \theta'$. Consider adding $n' > 0$ external candidates to the contest with $n > m$ internal candidates. Throughout, suppose that the contest with only the $n$ internal candidates produces challenge equilibria.

We show that, in any symmetric equilibrium of the expanded contest, the expanded contest has lower winner quality, measured by the probability that a selected contestant is strong, than the contest with only the $n$ internal candidates.

Consider the expanded contest. Let $\hat{p}_t$ and $\hat{p}'_t$ be the equilibrium probability of winning for a type-$t$ internal candidate and for a type-$t$ external candidate, respectively, in the expanded contest, $t \in \{S, W\}$. Because a weak internal candidate always has the option of mimicking a strong internal candidate’s strategy with probability $\mu_W/\mu_S$ and choosing zero performance with the complementary probability, it must be that

$$\hat{p}_W \geq \frac{\mu_W}{\mu_S} \hat{p}_S.$$  \hspace{1cm} (C-1)

Analogously,

$$\hat{p}'_W \geq \frac{\mu_W}{\mu_S} \hat{p}'_S.$$  \hspace{1cm} (C-2)
Let \( \hat{\Pi}(n, n') \) be winner quality after adding \( n' \) contestants to the original contest with \( n \) contestants. Note that

\[
\hat{\Pi}(n, n') = \frac{n\theta \hat{\rho}_S + n'\theta' \hat{\rho}'_S}{n\theta \hat{\rho}_S + n'\theta' \hat{\rho}'_S + n(1 - \theta)\hat{\rho}_W + n'(1 - \theta')\hat{\rho}'_W}
\]

where the equality follows from the fact that winner quality equals the expected number of strong winners divided by the sum of expected number of strong winners and the expected number of weak winners, and the inequality follows from (C-1), (C-2), and the fact that, for any fixed \( b > 0 \), \( f(a) = a/(a + b) \) is increasing in \( a \) for \( a > 0 \). Let \( \Pi(n) \) be winner quality in the original contest. Note that, if the original contest has challenge equilibria, then in the original contest, the probability that a weak contestant’s probability of winning equals \( p_0^G \) given by (10). Thus, by Bayes rule, the probability that a weak contestant is weak equals \( (1 - \theta)p_0^G/(m/n) \).

Hence, given that winner quality (i.e., the probability that a winner is strong) equals one minus the probability that a winner is weak, if the original contest has challenge equilibria,

\[
\Pi(n) = 1 - \frac{(1 - \theta)p_0^G}{m/n} = 1 - \frac{(1 - \theta)\mu_W}{\theta \mu_S + (1 - \theta)\mu_W} = \frac{\theta(\mu_S/\mu_W)}{\theta(\mu_S/\mu_W) + 1 - \theta}.
\]

where the second equality follows from equation (10).

By equations (C-3) and (C-4), if the original contest has challenge equilibria, then for any \( \theta' < \theta \) and \( n, n' > 0 \),

\[
\hat{\Pi}(n, n') - \Pi(n)
\]

where the result then follows immediately from the fact that, fixing \( m \), designer welfare (19) is maximized by maximizing winner quality.
**D. Generalized capacity constraints**

In the main text, we assumed that each type-\( t \in \{S, W\} \) contestant’s expected performance must equal the type-\( t \) contestant’s capacity, \( \mu_t \) (i.e., \( \mathbb{E}[X_t] = \mu_t \)), in which case risk-taking does not affect mean performance. In this appendix, we generalize the capacity constraint by imposing the mean constraint not on performance but on a continuous and increasing function, \( g : \mathbb{R}_+ \to \mathbb{R}_+ \), of performance, where \( g(0) = 0 \) and \( \lim_{x \to \infty} g(x) = \infty \). In particular, we assume that a type-\( t \) contestant’s choice of her random performance, \( X_t \), must satisfy that

\[
\mathbb{E}[g(X_t)] = g(\mu_t), \quad t = S, W. \tag{D-1}
\]

Under this specification, choosing a fixed performance level equal to \( \mu_t \) is feasible to a type-\( t \) contestant. Thus, \( \mu_t \) represents a type-\( t \) contestant’s performance if she takes no risk. If \( g \) is linear, we revert to our previous model, which imposes a mean constraint on performance. If \( g \) is strictly convex, then attaining high performance consumes disproportionately more capacity than attaining low performance. In this case, spreading out performance will require a reduction in mean performance in order to satisfy the generalized capacity constraint (D-1). Thus, strictly convex \( g \) captures situations in which increasing performance riskiness reduces expected performance.

For example, in Section 2, we assumed that a weak student, by taking no risk, receives 50 marks with certainty. Under our original specification of the capacity constraint, which requires this weak student’s mean performance to be fixed at 50, the performance distribution that gives the weak student 100 marks with half probability and zero marks with half probability is feasible to the weak student. In contrast, if \( g(x) = x^2 \), then this performance distribution will violate the generalized capacity constraint (D-1), and thus will no longer be feasible to the weak student.\(^{31}\) In fact, if \( g(x) = x^2 \), then to take extremely high risk by placing all weight on zero and 100 marks, the weak student’s chance of attaining 100 marks is only 25%, implying an expected mark equaling only 25 (despite the fact that, by playing safe, this weak student receives 50 marks).\(^{32}\)

It is also worth noting that contest situations in which contestants are restricted to risk-taking strategies with limited riskiness (e.g., variance) can be modeled under a generalized capacity constraint even without introducing an exogenous upper bound on performance. For example, when \( g(x) = x^2 \), the generalized capacity constraint implies that \( \mathbb{E}[X_t^2] \) equals a constant. Given the fact that \( \text{Var}[X_t] \leq \mathbb{E}[X_t^2] \), this constraint thus implies an upper bound on the variance of \( X_t \).

Generalizing the capacity constraint, using equation (D-1), in fact does not alter any of our qualitative conclusions on how changes of competitiveness affect meritocracy. To see this, note that, because \( g : \mathbb{R}_+ \to \mathbb{R}_+ \) is continuous and increasing, there is a 1-1 correspondence.

\[^{31}\text{This violation follows from the fact that } (1/2) \times 0^2 + (1/2) \times 100^2 = 5000 > 2500 = 50^2.\]

\[^{32}\text{The satisfaction of the constraint (D-1) follows from the fact that } (3/4) \times 0^2 + (1/4) \times 100^2 = 2500 = 50^2.\]
between performance, $x$, and transformed performance, $y = g(x)$. Because the transformation, $g$, is increasing, the ranking of transformed performance equals the ranking of performance. Thus, selection based on performance ranks is the same as selection based on transformed-performance ranks. The constraint (D-1) can be interpreted as a mean constraint imposed on transformed performance, and the space of transformed performance is $[0, \infty)$. Hence, if we think of contestants as competing by choosing random transformed-performance, such a competition is exactly the risk-taking contest we analyzed in the main text with a type-$t \in \{S, W\}$ contestant’s capacity equaling $g(\mu_t)$. Therefore, selection outcomes under the generalized capacity constraint (D-1) are the same as selection outcomes under our original capacity constraint with capacity being $g(\mu_t)$ for a type-$t \in \{S, W\}$ contestant. With both types’ capacities fixed, how changes of contest structure affect meritocracy follow from our analysis made in the main text.\footnote{In the main text, where we imposed the mean constraint on performance, the map from performance to the probability of winning in a symmetric equilibrium is given by $x \mapsto P(x)$. In contrast, this map, in our generalized capacity-constraint framework, does not characterize the relation between performance and the equilibrium probability of winning but characterizes the relation between transformed performance and the equilibrium probability of winning; the map from performance to the equilibrium probability of winning is instead given by $x \mapsto (P \circ g)(x)$.}